Mathematical Model for 3D Point-footed, Midleg-Mass, Hipped Walker without Yaw

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```
<< H:\Screws.m
<< H:\RobotLinks.m
```

```
Get::noopen : Cannot open LinearAlgebra`CrossProduct`. More…
```

```
Needs::nocont : Context LinearAlgebra`CrossProduct` was not created when Needs was evaluated. More…
```

Variables

```
x4 --> stance yaw angle
x5 --> stance roll angle
x6 --> stance pitch angle
x7 --> swing pitch angle
```

■ Constants

```
w --> hip width
l --> leg length
Mp --> pelvis / hip mass
M --> midleg mass
gamma --> slope angle
```

3D Walker Open Chain Kinematics

■ Foot/Leg links

```
q = {{x1[t]}, {x2[t]}, {x3[t]}, {x5[t]}, {x6[t]}, {x7[t]}};
qdot = D[q, t];

ax1 = {1, 0, 0, 0, 0, 0};
ax2 = {0, 1, 0, 0, 0, 0};
ax3 = {0, 0, 1, 0, 0, 0};
ax4 = {0, 0, 0, 0, 0, 1};
ax5 = {0, 0, 0, 0, 1, 0};
ax6 = {0, 0, 0, 1, 0, 0};
ax7 = Flatten[Append[Cross[{0, 0, l}, {1, 0, 0}], {1, 0, 0}]];

gst01 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, l / 2}, {0, 0, 0, 1}};
gst02 = {{1, 0, 0, w / 2}, {0, 1, 0, 0}, {0, 0, 1, l}, {0, 0, 0, 1}};
gst03 = {{1, 0, 0, w}, {0, 1, 0, 0}, {0, 0, 1, l / 2}, {0, 0, 0, 1}};
```

```
J1 = BodyJacobian[{ax1, x1[t]}, {ax2, x2[t]}, {ax3, x3[t]}, {ax5, x5[t]},
    {ax6, x6[t]}, {{0, 0, 0, 0, 0, 0}, x7[t]}, gst01] // FullSimplify;
J2 = BodyJacobian[{ax1, x1[t]}, {ax2, x2[t]}, {ax3, x3[t]}, {ax5, x5[t]},
    {ax6, x6[t]}, {{0, 0, 0, 0, 0, 0}, x7[t]}, gst02] // FullSimplify;
J3 = BodyJacobian[{ax1, x1[t]}, {ax2, x2[t]}, {ax3, x3[t]}, {ax5, x5[t]},
    {ax6, x6[t]}, {ax7, x7[t]}, gst03] // FullSimplify;
MatrixForm[J1]
MatrixForm[J2]
MatrixForm[J3]
```

$$\begin{pmatrix} \text{Cos}[x5[t]] & 0 & -\text{Sin}[x5[t]] & \frac{1}{2}\, l\, \text{Cos}[x6[t]] & 0 & 0 \\ \text{Sin}[x5[t]]\, \text{Sin}[x6[t]] & \text{Cos}[x6[t]] & \text{Cos}[x5[t]]\, \text{Sin}[x6[t]] & 0 & -\frac{l}{2} & 0 \\ \text{Cos}[x6[t]]\, \text{Sin}[x5[t]] & -\text{Sin}[x6[t]] & \text{Cos}[x5[t]]\, \text{Cos}[x6[t]] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \text{Cos}[x6[t]] & 0 & 0 \\ 0 & 0 & 0 & -\text{Sin}[x6[t]] & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[x5[t]] & 0 & -\text{Sin}[x5[t]] & l\, \text{Cos}[x6[t]] & 0 & 0 \\ \text{Sin}[x5[t]]\, \text{Sin}[x6[t]] & \text{Cos}[x6[t]] & \text{Cos}[x5[t]]\, \text{Sin}[x6[t]] & -\frac{1}{2}\, w\, \text{Sin}[x6[t]] & -l & 0 \\ \text{Cos}[x6[t]]\, \text{Sin}[x5[t]] & -\text{Sin}[x6[t]] & \text{Cos}[x5[t]]\, \text{Cos}[x6[t]] & -\frac{1}{2}\, w\, \text{Cos}[x6[t]] & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \text{Cos}[x6[t]] & 0 & 0 \\ 0 & 0 & 0 & -\text{Sin}[x6[t]] & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[x5[t]] & 0 & -\text{Sin}[x5[t]] & l\, \text{Cos}[x6 \\ \text{Sin}[x5[t]]\, \text{Sin}[x6[t]+x7[t]] & \text{Cos}[x6[t]+x7[t]] & \text{Cos}[x5[t]]\, \text{Sin}[x6[t]+x7[t]] & -w\, \text{Sin}[x \\ \text{Cos}[x6[t]+x7[t]]\, \text{Sin}[x5[t]] & -\text{Sin}[x6[t]+x7[t]] & \text{Cos}[x5[t]]\, \text{Cos}[x6[t]+x7[t]] & -w\, \text{Cos}[x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{Cos}[x6[\\ 0 & 0 & 0 & -\text{Sin}[x6 \end{pmatrix}$$

```
genM1 = M * {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0},
    {0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}};
genM2 = Mp * {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
    {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}};
genM3 = M * {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
    {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}};

MatrixForm[Mmatrix = Transpose[J1].genM1.J1 +
    Transpose[J2].genM2.J2 + Transpose[J3].genM3.J3 // FullSimplify]
```

$$\begin{pmatrix} 2\, M + Mp & 0 \\ 0 & 2\, M \\ 0 & 0 \\ \frac{1}{2}\, (l\, \text{Cos}[x5[t]]\, ((3\, M + 2\, Mp)\, \text{Cos}[x6[t]] - M\, \text{Cos}[x6[t]+x7[t]]) - (2\, M + Mp)\, w\, \text{Sin}[x5[t]]) & 0 \\ \frac{1}{2}\, l\, \text{Sin}[x5[t]]\, (-(3\, M + 2\, Mp)\, \text{Sin}[x6[t]] + M\, \text{Sin}[x6[t]+x7[t]]) & \frac{1}{2}\, l \\ \frac{1}{2}\, l\, M\, \text{Sin}[x5[t]]\, \text{Sin}[x6[t]+x7[t]] & \frac{1}{2}\, l \end{pmatrix}$$

```
MatrixForm[Mmatrix /. w → 0 // FullSimplify]
```

$$\begin{pmatrix} 2\,M + Mp & 0 \\ 0 & 2\,M + Mp \\ 0 & 0 \\ \frac{1}{2}\,l\,\text{Cos}[x5[t]]\,((3\,M + 2\,Mp)\,\text{Cos}[x6[t]] - M\,\text{Cos}[x6[t] + x7[t]]) & 0 \\ \frac{1}{2}\,l\,\text{Sin}[x5[t]]\,(-(3\,M + 2\,Mp)\,\text{Sin}[x6[t]] + M\,\text{Sin}[x6[t] + x7[t]]) & \frac{1}{2}\,l\,(-(3\,M + 2\,Mp)\,\text{Cos}[x6[t]] \\ \frac{1}{2}\,l\,M\,\text{Sin}[x5[t]]\,\text{Sin}[x6[t] + x7[t]] & \frac{1}{2}\,l\,M\,\text{Cos}[x6[t] + x7[t]] \end{pmatrix}$$

```
KE = First[First[1 / 2 Transpose[qdot].Mmatrix.qdot]] // FullSimplify
```

$$\begin{aligned}
&\frac{1}{16}\,(8\,(2\,M + Mp)\,x1'[t]^2 + 8\,(2\,M + Mp)\,x2'[t]^2 + 16\,M\,x3'[t]^2 + \\
&\quad 8\,Mp\,x3'[t]^2 - 16\,M\,w\,\text{Cos}[x5[t]]\,x3'[t]\,x5'[t] - 8\,Mp\,w\,\text{Cos}[x5[t]]\,x3'[t]\,x5'[t] - \\
&\quad 24\,l\,M\,\text{Cos}[x6[t]]\,\text{Sin}[x5[t]]\,x3'[t]\,x5'[t] - 16\,l\,Mp\,\text{Cos}[x6[t]]\,\text{Sin}[x5[t]]\,x3'[t]\,x5'[t] + \\
&\quad 8\,l\,M\,\text{Cos}[x6[t] + x7[t]]\,\text{Sin}[x5[t]]\,x3'[t]\,x5'[t] + 6\,l^2\,M\,x5'[t]^2 + \\
&\quad 4\,l^2\,Mp\,x5'[t]^2 + 8\,M\,w^2\,x5'[t]^2 + 2\,Mp\,w^2\,x5'[t]^2 + 5\,l^2\,M\,\text{Cos}[2\,x6[t]]\,x5'[t]^2 + \\
&\quad 4\,l^2\,Mp\,\text{Cos}[2\,x6[t]]\,x5'[t]^2 - 4\,l^2\,M\,\text{Cos}[x7[t]]\,x5'[t]^2 + \\
&\quad l^2\,M\,\text{Cos}[2\,(x6[t] + x7[t])]\,x5'[t]^2 - 4\,l^2\,M\,\text{Cos}[2\,x6[t] + x7[t]]\,x5'[t]^2 - \\
&\quad 24\,l\,M\,\text{Cos}[x5[t]]\,\text{Sin}[x6[t]]\,x3'[t]\,x6'[t] - 16\,l\,Mp\,\text{Cos}[x5[t]]\,\text{Sin}[x6[t]]\,x3'[t]\,x6'[t] + \\
&\quad 8\,l\,M\,\text{Cos}[x5[t]]\,\text{Sin}[x6[t] + x7[t]]\,x3'[t]\,x6'[t] + 16\,l\,M\,w\,\text{Sin}[x6[t]]\,x5'[t]\,x6'[t] + \\
&\quad 8\,l\,Mp\,w\,\text{Sin}[x6[t]]\,x5'[t]\,x6'[t] - 8\,l\,M\,w\,\text{Sin}[x6[t] + x7[t]]\,x5'[t]\,x6'[t] + \\
&\quad 12\,l^2\,M\,x6'[t]^2 + 8\,l^2\,Mp\,x6'[t]^2 - 8\,l^2\,M\,\text{Cos}[x7[t]]\,x6'[t]^2 - \\
&\quad 4\,l\,M\,(2\,\text{Sin}[x6[t] + x7[t]]\,(-\text{Cos}[x5[t]]\,x3'[t] + w\,x5'[t]) + l\,(-1 + 2\,\text{Cos}[x7[t]])\,x6'[t])\,x7'[t] + 2\,l^2\,M\,x7'[t]^2 + 8\,l\,x2'[t] \\
&\qquad ((-(3\,M + 2\,Mp)\,\text{Cos}[x6[t]] + M\,\text{Cos}[x6[t] + x7[t]])\,x6'[t] + M\,\text{Cos}[x6[t] + x7[t]]\,x7'[t]) + \\
&\quad 8\,x1'[t]\,((l\,\text{Cos}[x5[t]]\,((3\,M + 2\,Mp)\,\text{Cos}[x6[t]] - M\,\text{Cos}[x6[t] + x7[t]]) - \\
&\qquad (2\,M + Mp)\,w\,\text{Sin}[x5[t]])\,x5'[t] + l\,\text{Sin}[x5[t]] \\
&\qquad ((-(3\,M + 2\,Mp)\,\text{Sin}[x6[t]] + M\,\text{Sin}[x6[t] + x7[t]])\,x6'[t] + M\,\text{Sin}[x6[t] + x7[t]]\,x7'[t])))
\end{aligned}$$

```
KE /. {x1[t] → 0, x2[t] → 0, x3[t] → 0, x1'[t] → 0, x2'[t] → 0, x3'[t] → 0} // FullSimplify
```

$$\begin{aligned}
&\frac{1}{16}\,((l^2\,(6\,M + 4\,Mp) + 2\,(4\,M + Mp)\,w^2 + l^2\,((5\,M + 4\,Mp)\,\text{Cos}[2\,x6[t]] + \\
&\qquad M\,(-4\,\text{Cos}[x7[t]] + \text{Cos}[2\,(x6[t] + x7[t])] - 4\,\text{Cos}[2\,x6[t] + x7[t]])))\,x5'[t]^2 + 8\,l\,w \\
&\quad x5'[t]\,(((2\,M + Mp)\,\text{Sin}[x6[t]] - M\,\text{Sin}[x6[t] + x7[t]])\,x6'[t] - M\,\text{Sin}[x6[t] + x7[t]]\,x7'[t]) + \\
&\quad 2\,l^2\,((6\,M + 4\,Mp - 4\,M\,\text{Cos}[x7[t]])\,x6'[t]^2 + 2\,M\,(1 - 2\,\text{Cos}[x7[t]])\,x6'[t]\,x7'[t] + M\,x7'[t]^2))
\end{aligned}$$

■ Potential Term

```
MatrixForm[e1 = TwistExp[ax1, x1[t]]]
MatrixForm[e2 = TwistExp[ax2, x2[t]]]
MatrixForm[e3 = TwistExp[ax3, x3[t]]]
MatrixForm[e4 = TwistExp[ax4, x4[t]]]
MatrixForm[e5 = TwistExp[ax5, x5[t]]]
MatrixForm[e6 = TwistExp[ax6, x6[t]]]
MatrixForm[e7 = TwistExp[ax7, x7[t]]]
```

$$\begin{pmatrix} 1 & 0 & 0 & x1[t] \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & x2[t] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & x3[t] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[x4[t]] & -\text{Sin}[x4[t]] & 0 & 0 \\ \text{Sin}[x4[t]] & \text{Cos}[x4[t]] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[x5[t]] & 0 & \text{Sin}[x5[t]] & 0 \\ 0 & 1 & 0 & 0 \\ -\text{Sin}[x5[t]] & 0 & \text{Cos}[x5[t]] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Cos}[x6[t]] & -\text{Sin}[x6[t]] & 0 \\ 0 & \text{Sin}[x6[t]] & \text{Cos}[x6[t]] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Cos}[x7[t]] & -\text{Sin}[x7[t]] & l\,\text{Sin}[x7[t]] \\ 0 & \text{Sin}[x7[t]] & \text{Cos}[x7[t]] & l\,(1-\text{Cos}[x7[t]]) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
pS = {{0}, {0}, {l / 2}, {1}};
pH = {{w / 2}, {0}, {l}, {1}};
pNS = {{w}, {0}, {l / 2}, {1}};
```

```
MatrixForm[pSp = e1.e2.e3.e5.e6.pS // FullSimplify]
MatrixForm[pHp = e1.e2.e3.e5.e6.pH // FullSimplify]
MatrixForm[pNSp = e1.e2.e3.e5.e6.e7.pNS // FullSimplify]
```

$$\begin{pmatrix} \frac{1}{2}\, l \cos[x6[t]] \sin[x5[t]] + x1[t] \\ -\frac{1}{2}\, l \sin[x6[t]] + x2[t] \\ \frac{1}{2}\, l \cos[x5[t]] \cos[x6[t]] + x3[t] \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2}\, w \cos[x5[t]] + l \cos[x6[t]] \sin[x5[t]] + x1[t] \\ -l \sin[x6[t]] + x2[t] \\ l \cos[x5[t]] \cos[x6[t]] - \frac{1}{2}\, w \sin[x5[t]] + x3[t] \\ 1 \end{pmatrix}$$

```
General::spell :
 Possible spelling error: new symbol name "pNSp" is similar to existing symbols {pNS, pSp}. More…
```

$$\begin{pmatrix} w \cos[x5[t]] - \frac{1}{2}\, l\, (-2 \cos[x6[t]] + \cos[x6[t] + x7[t]]) \sin[x5[t]] + x1[t] \\ \frac{1}{2}\, l\, (-2 \sin[x6[t]] + \sin[x6[t] + x7[t]]) + x2[t] \\ -\frac{1}{2}\, l \cos[x5[t]]\, (-2 \cos[x6[t]] + \cos[x6[t] + x7[t]]) - w \sin[x5[t]] + x3[t] \\ 1 \end{pmatrix}$$

```
PE = First[M * g * pSp[[3]] + Mp * g * pHp[[3]] + M * g * pNSp[[3]]] // FullSimplify
```

$$\frac{1}{2}\, g\, (l \cos[x5[t]]\, ((3\, M + 2\, Mp) \cos[x6[t]] - M \cos[x6[t] + x7[t]]) - (2\, M + Mp)\, w \sin[x5[t]] + 2\, (2\, M + Mp)\, x3[t])$$

```
% /. w → 0
```

$$\frac{1}{2}\, g\, (l \cos[x5[t]]\, ((3\, M + 2\, Mp) \cos[x6[t]] - M \cos[x6[t] + x7[t]]) + 2\, (2\, M + Mp)\, x3[t])$$

3D Walker Dynamics

```
Lagrangian = KE - PE /.
    {x1[t] → 0, x2[t] → 0, x3[t] → 0, x1'[t] → 0, x2'[t] → 0, x3'[t] → 0} // FullSimplify;
```

```
eq1 = D[D[Lagrangian, x5'[t]], t] - D[Lagrangian, x5[t]] // FullSimplify
eq2 = D[D[Lagrangian, x6'[t]], t] - D[Lagrangian, x6[t]] // FullSimplify
eq3 = D[D[Lagrangian, x7'[t]], t] - D[Lagrangian, x7[t]] // FullSimplify
```

$$\frac{1}{16}$$
$$(-8\,g\,((2\,M+Mp)\,w\,\text{Cos}[x5[t]]+l\,((3\,M+2\,Mp)\,\text{Cos}[x6[t]]-M\,\text{Cos}[x6[t]+x7[t]])\,\text{Sin}[x5[t]])+$$
$$2\,l^2\,x5'[t]\,(-2\,(5\,M+4\,Mp)\,\text{Sin}[2\,x6[t]]\,x6'[t]+M\,(4\,\text{Sin}[x7[t]]\,x7'[t]-2\,\text{Sin}[$$
$$2\,(x6[t]+x7[t])]\,(x6'[t]+x7'[t])+4\,\text{Sin}[2\,x6[t]+x7[t]]\,(2\,x6'[t]+x7'[t])))+$$
$$2\,(l^2\,(6\,M+4\,Mp)+2\,(4\,M+Mp)\,w^2+l^2\,((5\,M+4\,Mp)\,\text{Cos}[2\,x6[t]]+$$
$$M\,(-4\,\text{Cos}[x7[t]]+\text{Cos}[2\,(x6[t]+x7[t])]-4\,\text{Cos}[2\,x6[t]+x7[t]])))\,x5''[t]+$$
$$8\,l\,w\,(((2\,M+Mp)\,\text{Cos}[x6[t]]-M\,\text{Cos}[x6[t]+x7[t]])\,x6'[t]^2-$$
$$2\,M\,\text{Cos}[x6[t]+x7[t]]\,x6'[t]\,x7'[t]-M\,\text{Cos}[x6[t]+x7[t]]\,x7'[t]^2+$$
$$((2\,M+Mp)\,\text{Sin}[x6[t]]-M\,\text{Sin}[x6[t]+x7[t]])\,x6''[t]-M\,\text{Sin}[x6[t]+x7[t]]\,x7''[t]))$$

$$\frac{1}{8}\,l\,(4\,g\,\text{Cos}[x5[t]]\,(-(3\,M+2\,Mp)\,\text{Sin}[x6[t]]+M\,\text{Sin}[x6[t]+x7[t]])+$$
$$l\,((5\,M+4\,Mp)\,\text{Sin}[2\,x6[t]]+M\,(\text{Sin}[2\,(x6[t]+x7[t])]-4\,\text{Sin}[2\,x6[t]+x7[t]]))\,x5'[t]^2+$$
$$8\,l\,M\,\text{Sin}[x7[t]]\,x6'[t]\,x7'[t]+$$
$$4\,(l\,M\,\text{Sin}[x7[t]]\,x7'[t]^2+w\,((2\,M+Mp)\,\text{Sin}[x6[t]]-M\,\text{Sin}[x6[t]+x7[t]])\,x5''[t]+$$
$$l\,(3\,M+2\,Mp-2\,M\,\text{Cos}[x7[t]])\,x6''[t])+2\,l\,M\,(1-2\,\text{Cos}[x7[t]])\,x7''[t])$$

$$-\frac{1}{8}\,l\,M\,(4\,\text{Sin}[x6[t]+x7[t]]\,(-g\,\text{Cos}[x5[t]]+w\,x5''[t])+$$
$$l\,((2\,\text{Sin}[x7[t]]-\text{Sin}[2\,(x6[t]+x7[t])]+2\,\text{Sin}[2\,x6[t]+x7[t]])\,x5'[t]^2+$$
$$4\,\text{Sin}[x7[t]]\,x6'[t]^2+(-2+4\,\text{Cos}[x7[t]])\,x6''[t]-2\,x7''[t]))$$

```
Solve[{eq1 == 0 , eq2 == 0, eq3 == 0}, {x5''[t], x6''[t], x7''[t]}] // FullSimplify
```

$$\{\{x5''[t]\to(-4\,g\,w\,\text{Cos}[x5[t]]\,((M^2+4\,M\,Mp+2\,Mp^2)\,\text{Cos}[2\,x6[t]]+$$
$$M\,(M+Mp)\,\text{Cos}[2\,x7[t]]+Mp\,(3\,M+2\,Mp+M\,\text{Cos}[2\,(x6[t]+x7[t])]))+$$
$$4\,g\,l\,(-3\,M-4\,Mp+2\,M\,\text{Cos}[2\,x7[t]])\,((3\,M+2\,Mp)\,\text{Cos}[x6[t]]-M\,\text{Cos}[x6[t]+x7[t]])$$
$$\text{Sin}[x5[t]]+l\,(w\,((M^2+7\,M\,Mp+4\,Mp^2)\,\text{Cos}[3\,x6[t]]+$$
$$\text{Cos}[x6[t]]\,(-21\,M^2-23\,M\,Mp-4\,Mp^2+M\,(5\,M+6\,Mp-2\,Mp\,\text{Cos}[2\,x6[t]])$$
$$\text{Cos}[x7[t]]+2\,M\,(9\,M+6\,Mp+(M+2\,Mp)\,\text{Cos}[2\,x6[t]])\,\text{Cos}[2\,x7[t]]-M$$
$$(3\,M-2\,Mp+2\,(M+3\,Mp)\,\text{Cos}[2\,x6[t]])\,\text{Cos}[3\,x7[t]])-4\,M\,(M+2\,Mp)\,\text{Cos}[x6[t]]^2$$
$$\text{Sin}[x6[t]]\,\text{Sin}[2\,x7[t]]+M\,\text{Sin}[x6[t]]\,((-13\,M-8\,Mp+2\,Mp\,\text{Cos}[2\,x6[t]])$$
$$\text{Sin}[x7[t]]+(5\,M+4\,Mp+2\,(M+3\,Mp)\,\text{Cos}[2\,x6[t]])\,\text{Sin}[3\,x7[t]]))\,x5'[t]^2+$$
$$2\,l\,(-3\,M-4\,Mp+2\,M\,\text{Cos}[2\,x7[t]])\,x5'[t]\,(((5\,M+4\,Mp)\,\text{Sin}[2\,x6[t]]+M$$
$$(\text{Sin}[2\,(x6[t]+x7[t])]-4\,\text{Sin}[2\,x6[t]+x7[t]]))\,x6'[t]+$$
$$M\,(-2\,\text{Sin}[x7[t]]+\text{Sin}[2\,(x6[t]+x7[t])]-2\,\text{Sin}[2\,x6[t]+x7[t]])\,x7'[t])+4\,w$$
$$((M^2+7\,M\,Mp+4\,Mp^2)\,\text{Cos}[x6[t]]\,x6'[t]^2+M\,(M+2\,Mp)\,\text{Cos}[x6[t]]\,\text{Cos}[2\,x7[t]]\,x6'[t]^2-$$
$$M\,(M+2\,Mp)\,\text{Sin}[x6[t]]\,\text{Sin}[2\,x7[t]]\,x6'[t]^2-M\,(M+4\,Mp)\,\text{Cos}[x6[t]]\,\text{Cos}[x7[t]]$$
$$(x6'[t]+x7'[t])^2+M\,(M+2\,Mp)\,\text{Sin}[x6[t]]\,\text{Sin}[x7[t]]\,(x6'[t]+x7'[t])^2)))/$$
$$(-2\,l^2\,(3\,M+2\,Mp)\,(3\,M+4\,Mp)-2\,(2\,M^2+7\,M\,Mp+2\,Mp^2)\,w^2+$$
$$\text{Cos}[2\,x6[t]]$$
$$(-2\,l^2\,(7\,M^2+16\,M\,Mp+8\,Mp^2)-4\,Mp\,(2\,M+Mp)\,w^2+$$
$$M\,(l^2\,(7\,M+4\,Mp)-4\,(M+2\,Mp)\,w^2)\,\text{Cos}[2\,x7[t]]+l^2\,M^2\,\text{Cos}[4\,x7[t]])+$$
$$M\,(4\,(l^2\,(3\,M+2\,Mp)-Mp\,w^2)\,\text{Cos}[2\,x7[t]]+8\,l^2\,\text{Cos}[x6[t]]^2$$
$$(2\,(M+2\,Mp)\,\text{Cos}[x7[t]]-M\,\text{Cos}[3\,x7[t]])+$$
$$2\,(2\,(M+2\,Mp)\,(l^2+2\,w^2)\,\text{Cos}[x7[t]]-l^2\,(6\,M+8\,Mp-4\,M\,\text{Cos}[2\,x7[t]]+M\,\text{Cos}[3\,x7[t]]))$$
$$\text{Sin}[2\,x6[t]]\,\text{Sin}[x7[t]])),$$
$$x6''[t]\to-(4\,g\,(-\text{Cos}[x5[t]]\,((l^2\,(3\,M+2\,Mp)\,(5\,M+4\,Mp)+8\,M\,(M+Mp)\,w^2-$$
$$2\,M\,(l^2\,(6\,M+4\,Mp)-2\,Mp\,w^2+(l^2\,(3\,M+2\,Mp)-4\,(M+Mp)\,w^2)\,\text{Cos}[2\,x6[t]])$$
$$\text{Cos}[2\,x7[t]]+16\,l^2\,M\,\text{Cos}[x6[t]]^2\,(-2\,(M+Mp)\,\text{Cos}[x7[t]]+M\,\text{Cos}[3\,x7[t]]))$$

```
Sin[x6[t]] - l^2 (-2 (M + Mp) (5 M + 4 Mp) + M^2 Cos[4 x7[t]]) Sin[3 x6[t]] +
M (4 Cos[x6[t]] (-2 (l^2 (4 M + 3 Mp) + (2 M + Mp) w^2) Cos[x7[t]] +
Cos[2 x6[t]] (-4 l^2 (M + Mp) + (-l^2 (3 M + 2 Mp) + 4 (M + Mp) w^2) Cos[x7[t]]) +
l^2 (7 M + 4 Mp + 2 M Cos[2 x7[t]])) Sin[x7[t]] -
l^2 M Cos[3 x6[t]] (-4 Sin[3 x7[t]] + Sin[4 x7[t]]))) + 4 l w Sin[x5[t]]
((M + Mp) (3 M + 2 Mp) Sin[2 x6[t]] + M ((2 M + Mp) Sin[x7[t]] - (3 M + 2 Mp)
Sin[2 x7[t]] - 3 M Sin[2 (x6[t] + x7[t])] - 2 Mp Sin[2 (x6[t] + x7[t])] -
M Sin[2 x6[t] + x7[t]] - Mp Sin[2 x6[t] + x7[t]] + M Sin[2 x6[t] + 3 x7[t]]))) +
l ((2 (l^2 (25 M^2 + 36 M Mp + 16 Mp^2) + 2 (10 M^2 + 15 M Mp + 4 Mp^2) w^2 + M (-(l^2 (19 M + 20 Mp) +
4 (2 M + Mp) w^2 + l^2 (17 M + 20 Mp) Cos[2 x6[t]]) Cos[x7[t]] - (8 l^2 (2 M +
Mp) + 4 (4 M + Mp) w^2 + (l^2 (3 M + 4 Mp) - 4 (M + 2 Mp) w^2) Cos[2 x6[t]])
Cos[2 x7[t]] + (l^2 (15 M + 4 Mp) + 8 M w^2 + 4 (l^2 (3 M + Mp) + Mp w^2)
Cos[2 x6[t]]) Cos[3 x7[t]] - 3 l^2 M Cos[4 x7[t]])) Sin[2 x6[t]] +
l^2 ((3 M + 4 Mp) (5 M + 4 Mp) + M^2 (-6 Cos[4 x7[t]] + Cos[5 x7[t]])) Sin[4 x6[t]] +
2 M (l^2 (31 M + 16 Mp) + 2 (8 M + 3 Mp) w^2 - 16 (5 M + 3 Mp) (l^2 + w^2) Cos[x6[t]]^2
Cos[x7[t]] + l^2 (13 M + 4 Mp) Cos[2 x7[t]] + 2 Cos[2 x6[t]] (l^2 (13 M + 4 Mp) -
2 Mp w^2 + (l^2 (15 M + 4 Mp) + 8 M w^2) Cos[2 x7[t]] - 2 Cos[x6[t]]^2
Cos[x7[t]] (l^2 (3 M + 4 Mp) - 4 (M + 2 Mp) w^2 + 12 l^2 M Cos[2 x7[t]])) +
12 l^2 M Cos[x6[t]]^2 Cos[3 x7[t]] + Cos[4 x6[t]] (-2 l^2 (M + 4 Mp) + 2 Mp w^2 +
(l^2 (13 M + 4 Mp) + 4 Mp w^2) Cos[2 x7[t]] + l^2 M Cos[4 x7[t]])) Sin[x7[t]])
x5'[t]^2 + 16 l w ((M + Mp) Sin[x6[t]] - M Sin[x6[t] + 2 x7[t]]) x5'[t]
(((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]])) x6'[
t] + M (-2 Sin[x7[t]] + Sin[2 (x6[t] + x7[t])] - 2 Sin[2 x6[t] + x7[t]]) x7'[t]) +
4 ((-(-5 l^2 M^2 + 4 Mp (2 M + Mp) w^2 + M (5 l^2 M - 4 Mp w^2) Cos[x7[t]] +
l^2 M^2 (4 Cos[2 x7[t]] - 5 Cos[3 x7[t]] + Cos[4 x7[t]])) Sin[2 x6[t]] +
2 M (4 (-2 l^2 (2 M + Mp) + Mp w^2) Cos[x7[t]] + 2 l^2 (5 M + 2 Mp + 2 M Cos[2 x7[t]]) +
Cos[2 x6[t]] (l^2 (9 M + 4 Mp) + 2 Mp w^2 - l^2 ((15 M + 8 Mp) Cos[x7[t]] +
M (-5 Cos[2 x7[t]] + Cos[3 x7[t]])))) Sin[x7[t]]) x6'[t]^2 +
2 M (4 l^2 M Sin[2 x6[t]] - l^2 (5 M + 4 Mp) Sin[2 x6[t] - x7[t]] +
4 (l^2 (M + Mp) + Mp w^2) Sin[2 x6[t] + x7[t]] +
4 l^2 ((3 M + 2 Mp) Sin[x7[t]] - 2 M Cos[x6[t]] Sin[x6[t] + 2 x7[t]]) +
l^2 M Sin[2 x6[t] + 3 x7[t]]) x6'[t] x7'[t] + M (4 l^2 M Sin[2 x6[t]] - l^2
(5 M + 4 Mp) Sin[2 x6[t] - x7[t]] + 4 (l^2 (M + Mp) + Mp w^2) Sin[2 x6[t] + x7[t]] +
4 l^2 ((3 M + 2 Mp) Sin[x7[t]] - 2 M Cos[x6[t]] Sin[x6[t] + 2 x7[t]]) +
l^2 M Sin[2 x6[t] + 3 x7[t]]) x7'[t]^2))) /
(4 l (Cos[2 x6[t]] (2 l^2 (7 M^2 + 16 M Mp + 8 Mp^2) + 4 Mp (2 M + Mp) w^2 +
M (-l^2 (7 M + 4 Mp) + 4 (M + 2 Mp) w^2) Cos[2 x7[t]] - l^2 M^2 Cos[4 x7[t]]) +
2 (l^2 (3 M + 2 Mp) (3 M + 4 Mp) + (2 M^2 + 7 M Mp + 2 Mp^2) w^2 + 2 M (-l^2 (3 M + 2 Mp) + Mp w^2)
Cos[2 x7[t]] + 4 l^2 M Cos[x6[t]]^2 (-2 (M + 2 Mp) Cos[x7[t]] + M Cos[3 x7[t]]) +
M (-2 (M + 2 Mp) (l^2 + 2 w^2) Cos[x7[t]] + l^2 (6 M + 8 Mp - 4 M Cos[2 x7[t]] +
M Cos[3 x7[t]])) Sin[2 x6[t]] Sin[x7[t]]))),
x7''[t] → (4 g (4 l w Sin[x5[t]] ((5 M^2 + 6 M Mp + 2 Mp^2 + (2 M^2 + 7 M Mp + 4 Mp^2)
Cos[x7[t]] - M (6 M + 5 Mp) Cos[2 x7[t]] + M^2 Cos[3 x7[t]]) Sin[2 x6[t]] +
(17 M^2 + 23 M Mp + 8 Mp^2 - 2 M (5 M + 3 Mp) Cos[x7[t]] + Cos[2 x6[t]] (15 M^2 +
21 M Mp + 8 Mp^2 - 2 M (6 M + 5 Mp) Cos[x7[t]] + 2 M^2 Cos[2 x7[t]])) Sin[x7[t]]) +
Cos[x5[t]] ((-l^2 (3 M + 2 Mp) (9 M + 4 Mp) - 8 M (M + Mp) w^2 + (l^2 M (17 M + 14 Mp) +
4 (2 M^2 + 2 M Mp + Mp^2) w^2) Cos[x7[t]] + M (l^2 (17 M + 14 Mp) + 4 M w^2)
Cos[2 x7[t]] - l^2 M (7 M + 2 Mp) Cos[3 x7[t]]) Sin[x6[t]] +
(2 (l^2 M (8 M + 7 Mp) + 2 (M + Mp) (2 M + Mp) w^2) Cos[x7[t]] + M
(l^2 (11 M + 10 Mp) - 4 (M + Mp) w^2) Cos[2 x7[t]] - l^2 (22 M^2 + 26 M Mp +
8 Mp^2 + 2 M (3 M + Mp) Cos[3 x7[t]] - M^2 Cos[4 x7[t]])) Sin[3 x6[t]] -
Cos[x6[t]] (l^2 (85 M^2 + 104 M Mp + 32 Mp^2) + 4 (4 M^2 + M Mp - 2 Mp^2) w^2 + 4 (M + Mp)
(l^2 (7 M + 8 Mp) - 2 (2 M + Mp) w^2) Cos[2 x6[t]]) Sin[x7[t]] + 2 M Cos[x6[t]]
```

```
          (2 l^2 (10 M + 7 Mp) + 2 (2 M + Mp) w^2 + (l^2 (11 M + 10 Mp) - 4 (M + Mp) w^2) Cos[2 x6[t]])
          Sin[2 x7[t]] - l^2 M Cos[x6[t]] (M + 4 (3 M + Mp) Cos[2 x6[t]])
          Sin[3 x7[t]] + l^2 M^2 Cos[3 x6[t]] Sin[4 x7[t]])) +
     l ((2 (2 l^2 (29 M^2 + 30 M Mp + 8 Mp^2) + 2 (2 M + Mp) (9 M + 4 Mp) w^2 - 2 (l^2 M (15 M + 8 Mp) -
              4 Mp (2 M + Mp) w^2 + (3 l^2 M (5 M + 4 Mp) + 2 (2 M + Mp) (M + 2 Mp) w^2) Cos[2 x6[t]])
          Cos[x7[t]] - 2 (l^2 (3 M + 2 Mp) (9 M + 4 Mp) + (20 M^2 + 17 M Mp + 2 Mp^2) w^2 +
            2 (M + Mp) (l^2 (9 M + 4 Mp) + (-M + Mp) w^2) Cos[2 x6[t]]) Cos[2 x7[t]] + M
          (2 l^2 (15 M + 8 Mp) + 8 M w^2 + (l^2 (29 M + 24 Mp) + 4 Mp w^2) Cos[2 x6[t]])
          Cos[3 x7[t]] - l^2 M (4 M + (9 M + 4 Mp) Cos[2 x6[t]]) Cos[4 x7[t]]) Sin[2 x6[t]] +
        l^2 ((5 M + 4 Mp) (9 M + 4 Mp) + M^2 Cos[5 x7[t]]) Sin[4 x6[t]] +
        2 (2 l^2 (67 M^2 + 76 M Mp + 24 Mp^2) + 20 (5 M^2 + 5 M Mp + Mp^2) w^2 - 2 (l^2 (3 M + 2 Mp)
              (25 M + 4 Mp) + 2 (17 M^2 + 8 M Mp + Mp^2) w^2) Cos[x7[t]] + 2 l^2 M (13 M + 4 Mp)
          Cos[2 x7[t]] + 4 Cos[2 x6[t]] (l^2 (39 M^2 + 48 M Mp + 16 Mp^2) + 4 (5 M^2 + 5 M Mp + Mp^2)
              w^2 - (l^2 (49 M^2 + 46 M Mp + 8 Mp^2) + (20 M^2 + 17 M Mp + 2 Mp^2) w^2) Cos[x7[t]] +
            M ((l^2 (15 M + 8 Mp) + 4 M w^2) Cos[2 x7[t]] - 2 l^2 M Cos[3 x7[t]])) +
          Cos[4 x6[t]] (l^2 (5 M + 4 Mp)^2 - 4 (M + Mp)^2 w^2 - (l^2 (5 M + 4 Mp) (9 M + 4 Mp) +
              4 (-M^2 + Mp^2) w^2) Cos[x7[t]] + M ((6 l^2 (5 M + 4 Mp) + 4 Mp w^2) Cos[2 x7[t]] +
              l^2 (-(9 M + 4 Mp) Cos[3 x7[t]] + M Cos[4 x7[t]])))) Sin[x7[t]]) x5'[t]^2 +
     16 l w ((M + Mp + (M + 2 Mp) Cos[x7[t]] - M Cos[2 x7[t]]) Sin[x6[t]] +
        Cos[x6[t]] (5 M + 4 Mp - 2 M Cos[x7[t]]) Sin[x7[t]]) x5'[t]
      (((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]])) x6'[
          t] + M (-2 Sin[x7[t]] + Sin[2 (x6[t] + x7[t])] - 2 Sin[2 x6[t] + x7[t]]) x7'[t]) +
     4 (2 (-(-l^2 M (13 M + 8 Mp) + 2 Mp (2 M + Mp) w^2 + (l^2 M (7 M + 2 Mp) + 4 (M + Mp)^2 w^2)
              Cos[x7[t]] + 2 M (l^2 (6 M + 4 Mp) - (M + 2 Mp) w^2) Cos[2 x7[t]] +
            l^2 M (-(7 M + 2 Mp) Cos[3 x7[t]] + M Cos[4 x7[t]]))
          Sin[2 x6[t]] + 2 (l^2 (22 M^2 + 24 M Mp + 8 Mp^2) + 2 (M - Mp) (M + Mp) w^2 +
            4 M ((-2 l^2 (3 M + 2 Mp) + Mp w^2) Cos[x7[t]] + l^2 M Cos[2 x7[t]]) +
            Cos[2 x6[t]] (l^2 (19 M^2 + 22 M Mp + 8 Mp^2) - 2 (M + Mp)^2 w^2 +
              M (-l^2 (23 M + 16 Mp) + 2 (M + 2 Mp) w^2) Cos[x7[t]] +
              l^2 M ((7 M + 2 Mp) Cos[2 x7[t]] - M Cos[3 x7[t]]))) Sin[x7[t]]) x6'[t]^2 +
        2 M (((-5 l^2 M + 4 Mp w^2) Cos[x7[t]] + (-4 l^2 M + 4 (M + 2 Mp) w^2) Cos[2 x7[t]] +
            l^2 M (5 + 5 Cos[3 x7[t]] - Cos[4 x7[t]])) Sin[2 x6[t]] + 2 (4 (-2 l^2 (2 M + Mp) +
              Mp w^2) Cos[x7[t]] + 2 l^2 (5 M + 2 Mp + 2 M Cos[2 x7[t]]) + Cos[2 x6[t]]
            (l^2 (9 M + 4 Mp) + 2 Mp w^2 + (-l^2 (15 M + 8 Mp) + 4 (M + 2 Mp) w^2) Cos[x7[t]] -
              l^2 M (-5 Cos[2 x7[t]] + Cos[3 x7[t]]))) Sin[x7[t]]) x6'[t] x7'[t] +
        M (((-5 l^2 M + 4 Mp w^2) Cos[x7[t]] + (-4 l^2 M + 4 (M + 2 Mp) w^2) Cos[2 x7[t]] +
            l^2 M (5 + 5 Cos[3 x7[t]] - Cos[4 x7[t]])) Sin[2 x6[t]] + 2 (4 (-2 l^2 (2 M + Mp) +
              Mp w^2) Cos[x7[t]] + 2 l^2 (5 M + 2 Mp + 2 M Cos[2 x7[t]]) + Cos[2 x6[t]]
            (l^2 (9 M + 4 Mp) + 2 Mp w^2 + (-l^2 (15 M + 8 Mp) + 4 (M + 2 Mp) w^2) Cos[x7[t]] -
              l^2 M (-5 Cos[2 x7[t]] + Cos[3 x7[t]]))) Sin[x7[t]]) x7'[t]^2))) /
   (4 l (Cos[2 x6[t]] (2 l^2 (7 M^2 + 16 M Mp + 8 Mp^2) + 4 Mp (2 M + Mp) w^2 +
          M (-l^2 (7 M + 4 Mp) + 4 (M + 2 Mp) w^2) Cos[2 x7[t]] - l^2 M^2 Cos[4 x7[t]]) +
        2 (l^2 (3 M + 2 Mp) (3 M + 4 Mp) + (2 M^2 + 7 M Mp + 2 Mp^2) w^2 +
          2 M (-l^2 (3 M + 2 Mp) + Mp w^2) Cos[2 x7[t]] +
          4 l^2 M Cos[x6[t]]^2 (-2 (M + 2 Mp) Cos[x7[t]] + M Cos[3 x7[t]]) +
          M (-2 (M + 2 Mp) (l^2 + 2 w^2) Cos[x7[t]] +
            l^2 (6 M + 8 Mp - 4 M Cos[2 x7[t]] + M Cos[3 x7[t]])) Sin[2 x6[t]] Sin[x7[t]])))}}
```

- **Without a hip:**

```
Solve[{eq1 == 0 /. w → 0, eq2 == 0 /. w → 0, eq3 == 0 /. w → 0},
  {x5''[t], x6''[t], x7''[t]}] // FullSimplify
```

$$\left\{\left\{x5''[t] \to \right.\right.$$
$$(2\,(2\,g\,((3\,M+2\,Mp)\,\text{Cos}[x6[t]]-M\,\text{Cos}[x6[t]+x7[t]])\,\text{Sin}[x5[t]]+l\,x5'[t]\,(((5\,M+4\,Mp)\,\text{Sin}[2\,x6[t]]+M\,(\text{Sin}[2\,(x6[t]+x7[t])]-4\,\text{Sin}[2\,x6[t]+x7[t]]))\,x6'[t]+M\,(-2\,\text{Sin}[x7[t]]+\text{Sin}[2\,(x6[t]+x7[t])]-2\,\text{Sin}[2\,x6[t]+x7[t]])\,x7'[t])))/$$
$$(l\,(6\,M+4\,Mp+(5\,M+4\,Mp)\,\text{Cos}[2\,x6[t]]+M\,(-4\,\text{Cos}[x7[t]]+\text{Cos}[2\,(x6[t]+x7[t])]-4\,\text{Cos}[2\,x6[t]+x7[t]]))),$$
$$x6''[t] \to \frac{1}{2\,l\,(-3\,M-4\,Mp+2\,M\,\text{Cos}[2\,x7[t]])}$$
$$(4\,g\,\text{Cos}[x5[t]]\,(-2\,(M+Mp)\,\text{Sin}[x6[t]]+M\,\text{Sin}[x6[t]+2\,x7[t]])+l\,((3\,M+4\,Mp)\,\text{Sin}[2\,x6[t]]+4\,M\,(\text{Cos}[x6[t]+x7[t]]^2\,\text{Sin}[x7[t]]-\text{Cos}[x6[t]]\,\text{Sin}[x6[t]+2\,x7[t]]))\,x5'[t]^2+4\,l\,M\,(-\text{Sin}[2\,x7[t]]\,x6'[t]^2+\text{Sin}[x7[t]]\,(x6'[t]+x7'[t])^2)),$$
$$x7''[t] \to \frac{1}{l\,(-3\,M-4\,Mp+2\,M\,\text{Cos}[2\,x7[t]])}$$
$$(2\,g\,\text{Cos}[x5[t]]\,(-(3\,M+2\,Mp)\,(-1+2\,\text{Cos}[x7[t]])\,\text{Sin}[x6[t]]+(5\,M+4\,Mp-2\,M\,\text{Cos}[x7[t]])\,\text{Sin}[x6[t]+x7[t]])+l\,\text{Sin}[x7[t]]\,(-(6\,M+4\,Mp+(5\,M+4\,Mp)\,\text{Cos}[2\,x6[t]]-4\,M\,\text{Cos}[x7[t]]+M\,\text{Cos}[2\,(x6[t]+x7[t])]-(5\,M+4\,Mp)\,\text{Cos}[2\,x6[t]+x7[t]])\,x5'[t]^2+4\,(-3\,M-2\,Mp+2\,M\,\text{Cos}[x7[t]])\,x6'[t]^2+4\,M\,(-1+2\,\text{Cos}[x7[t]])\,x6'[t]\,x7'[t]+2\,M\,(-1+2\,\text{Cos}[x7[t]])\,x7'[t]^2))\Big\}\Big\}$$

Impact Equations

```
pNSFoot = {{w}, {0}, {0}, {1}};
MatrixForm[pNSFootp = e1.e2.e3.e5.e6.e7.pNSFoot // FullSimplify]
```

$$\begin{pmatrix} w\,\text{Cos}[x5[t]]+2\,l\,\text{Sin}[x5[t]]\,\text{Sin}[x6[t]+\frac{x7[t]}{2}]\,\text{Sin}[\frac{x7[t]}{2}]+x1[t] \\ l\,(-\text{Sin}[x6[t]]+\text{Sin}[x6[t]+x7[t]])+x2[t] \\ -w\,\text{Sin}[x5[t]]+2\,l\,\text{Cos}[x5[t]]\,\text{Sin}[x6[t]+\frac{x7[t]}{2}]\,\text{Sin}[\frac{x7[t]}{2}]+x3[t] \\ 1 \end{pmatrix}$$

```
Pos1 = First[pNSFootp[[1]]];
Pos2 = First[pNSFootp[[2]]];
Pos3 = First[pNSFootp[[3]]];
MatrixForm[F = FullSimplify[
   {{D[Pos1, x1[t]], D[Pos1, x2[t]],
     D[Pos1, x3[t]], D[Pos1, x5[t]], D[Pos1, x6[t]], D[Pos1, x7[t]]},
    {D[Pos2, x1[t]], D[Pos2, x2[t]], D[Pos2, x3[t]], D[Pos2, x5[t]],
     D[Pos2, x6[t]], D[Pos2, x7[t]]},
    {D[Pos3, x1[t]], D[Pos3, x2[t]], D[Pos3, x3[t]], D[Pos3, x5[t]],
     D[Pos3, x6[t]], D[Pos3, x7[t]]}}]]
```

$$\begin{pmatrix} 1 & 0 & 0 & -w\,\text{Sin}[x5[t]]+2\,l\,\text{Cos}[x5[t]]\,\text{Sin}[x6[t]+\frac{x7[t]}{2}]\,\text{Sin}[\frac{x7[t]}{2}] & 2\,l\,\text{Cos}[x6[t]+\frac{x7[t]}{2}]\,S \\ 0 & 1 & 0 & 0 & l\,(-\text{Cos}[x6[t]]+\text{Cos}[x \\ 0 & 0 & 1 & -w\,\text{Cos}[x5[t]]-2\,l\,\text{Sin}[x5[t]]\,\text{Sin}[x6[t]+\frac{x7[t]}{2}]\,\text{Sin}[\frac{x7[t]}{2}] & 2\,l\,\text{Cos}[x5[t]]\,\text{Cos}[x6[\end{pmatrix}$$

```
MatrixForm[nFT = Join[-Transpose[F], {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}]]
```

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ w\,\text{Sin}[x5[t]] - 2\,l\,\text{Cos}[x5[t]]\,\text{Sin}[x6[t] + \frac{x7[t]}{2}]\,\text{Sin}[\frac{x7[t]}{2}] & 0 \\ -2\,l\,\text{Cos}[x6[t] + \frac{x7[t]}{2}]\,\text{Sin}[x5[t]]\,\text{Sin}[\frac{x7[t]}{2}] & -l\,(-\text{Cos}[x6[t]] + \text{Cos}[x6[t] + x7[\\ -l\,\text{Sin}[x5[t]]\,\text{Sin}[x6[t] + x7[t]] & -l\,\text{Cos}[x6[t] + x7[t]] \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
MatrixForm[FullMatrix = Transpose[Join[Transpose[Join[Mmatrix, F]], Transpose[nFT]]]]
```

$$\begin{pmatrix} 2\,M + Mp & 0 \\ 0 & 2\,M \\ 0 & 0 \\ \frac{1}{2}\,(l\,\text{Cos}[x5[t]]\,((3\,M + 2\,Mp)\,\text{Cos}[x6[t]] - M\,\text{Cos}[x6[t] + x7[t]]) - (2\,M + Mp)\,w\,\text{Sin}[x5[t]]) & 0 \\ \frac{1}{2}\,l\,\text{Sin}[x5[t]]\,(-(3\,M + 2\,Mp)\,\text{Sin}[x6[t]] + M\,\text{Sin}[x6[t] + x7[t]]) & \frac{1}{2}\,l \\ \frac{1}{2}\,l\,M\,\text{Sin}[x5[t]]\,\text{Sin}[x6[t] + x7[t]] & \frac{1}{2}\,l \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

```
MatrixForm[FullMatrix /. w → 0 // FullSimplify]
```

$$\begin{pmatrix} 2\,M + Mp & 0 \\ 0 & 2\,M + Mp \\ 0 & 0 \\ \frac{1}{2}\,l\,\text{Cos}[x5[t]]\,((3\,M + 2\,Mp)\,\text{Cos}[x6[t]] - M\,\text{Cos}[x6[t] + x7[t]]) & 0 \\ \frac{1}{2}\,l\,\text{Sin}[x5[t]]\,(-(3\,M + 2\,Mp)\,\text{Sin}[x6[t]] + M\,\text{Sin}[x6[t] + x7[t]]) & \frac{1}{2}\,l\,(-(3\,M + 2\,Mp)\,\text{Cos}[x6[t]] \\ \frac{1}{2}\,l\,M\,\text{Sin}[x5[t]]\,\text{Sin}[x6[t] + x7[t]] & \frac{1}{2}\,l\,M\,\text{Cos}[x6[t] + x7[t]] \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

```
MatrixForm[DMatrix = Join[Mmatrix.{0, 0, 0, x5'[t], x6'[t], x7'[t]}, {0, 0, 0}]]
```

$$\begin{pmatrix} \frac{1}{2}\,(l\,\text{Cos}[x5[t]]\,((3\,M + 2\,Mp)\,\text{Cos}[x6[t]] - M\,\text{Cos}[x6[t] + x7[t]]) - (2\,M + Mp)\,w\,\text{Sin}[x5[t]])\,x5'[t \\ \frac{1}{2}\,l\,(-(3\,M + 2\,Mp)\,\text{Cos}[x6[t]] + M\,\text{Cos}[x6[t] + x7[t]])\,x6'[t] + \frac{1}{2}\,l\,M\,\text{Cos}[x6[t] + x7[t]]\,x7'[t] \\ \frac{1}{2}\,(-(2\,M + Mp)\,w\,\text{Cos}[x5[t]] + l\,(-(3\,M + 2\,Mp)\,\text{Cos}[x6[t]] + M\,\text{Cos}[x6[t] + x7[t]])\,\text{Sin}[x5[t]])\,x5' \\ \frac{1}{8}\,(l^2\,(6\,M + 4\,Mp) + 2\,(4\,M + Mp)\,w^2 + l^2\,((5\,M + 4\,Mp)\,\text{Cos}[2\,x6[t]] + M\,(-4\,\text{Cos}[x7[t]] + \text{Cos}[2\,(x6[t] \\ \frac{1}{2}\,l\,w\,((2\,M + Mp)\,\text{Sin}[x6[t]] - M\,\text{Sin}[x6[t] + x7[t]])\,x5'[t] + \frac{1}{2}\,l^2\,(3\,M + 2\,Mp - 2\,M\,\text{Cos}[x7[t]])\,x6' \\ -\frac{1}{2}\,l\,M\,w\,\text{Sin}[x6[t] + x7[t]]\,x5'[t] + \frac{1}{4}\,l\,M\,(l - 2\,l\,\text{Cos}[x7[t]])\,x6'[t] + \frac{1}{4}\,l^2\,M\,x7'[t] \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
(% FullMatrix = FullMatrix /. w → 0 // FullSimplify;
 DMatrix = DMatrix /. w → 0 // FullSimplify; %)

K = Inverse[FullMatrix].DMatrix;
```

First, we have the angular positions for 5,6,7:

```
{x5impact = x5[t], x6impact = x6[t] + x7[t], x7impact = -x7[t]};
```

We are interested in the angular velocities for 5,6,7:

```
x5dotimpact = FullSimplify[K[[4]]]
x6dotimpact = FullSimplify[K[[5]] + K[[6]]]
x7dotimpact = -FullSimplify[K[[6]]]
```

$$-\Big(\big(\text{Cos}[2\,x6[t]]\,\big(\big(l^2\,M\,(M+4\,Mp)+4\,(M+Mp)^2\,w^2\big)\,\text{Cos}[2\,x7[t]]-M\,\big(-2\,l^2\,(M+2\,Mp)+4\,(M+Mp)\,w^2+l^2\,M\,\text{Cos}[4\,x7[t]]\big)\big)+2\,\big(l^2\,M\,(3\,M+4\,Mp)-\big(2\,M^2+M\,Mp-2\,Mp^2\big)\,w^2+2\,M^2\,\big(-l^2+w^2\big)\,\text{Cos}[2\,x7[t]]+4\,l^2\,(M+Mp)\,\text{Cos}[x6[t]]^2\,\text{Cos}[x7[t]]\,(-3\,M-4\,Mp+2\,M\,\text{Cos}[2\,x7[t]])+\big(-2\,\big(l^2\,M\,(M+2\,Mp)+2\,(M+Mp)^2\,w^2\big)\,\text{Cos}[x7[t]]+l^2\,\big(-2\,(M+Mp)\,(-3\,M-4\,Mp+2\,M\,\text{Cos}[2\,x7[t]])+M^2\,\text{Cos}[3\,x7[t]]\big)\big)\,\text{Sin}[2\,x6[t]]\,\text{Sin}[x7[t]]\big)\,x5'[t]+4\,l\,w\,\Big(\Big(2\,Mp\,(5\,M+4\,Mp+(6\,M+4\,Mp)\,\text{Cos}[x7[t]])\,\text{Sin}[x6[t]]\,\text{Sin}\Big[\frac{x7[t]}{2}\Big]^2-(2\,M+Mp)\,\text{Cos}[x6[t]]\,\big(-M+2\,(M+2\,Mp)\,\text{Cos}[x7[t]]\big)\,\text{Sin}[x7[t]]\Big)\,x6'[t]+M\,\big(2\,M\,\text{Cos}[x6[t]]\,\text{Sin}[x7[t]]+Mp\,\text{Sin}[x6[t]+x7[t]]\big)\,x7'[t]\Big)\Big)\Big/\big(\text{Cos}[2\,x6[t]]\,\big(\big(l^2\,(13\,M^2+32\,M\,Mp+16\,Mp^2)+4\,Mp\,(2\,M+Mp)\,w^2\big)\,\text{Cos}[2\,x7[t]]+M\,\big(-2\,l^2\,M+4\,(M+2\,Mp)\,w^2-l^2\,(5\,M+4\,Mp)\,\text{Cos}[4\,x7[t]]\big)\big)+2\,\big(l^2\,(3\,M+2\,Mp)\,(3\,M+4\,Mp)+\big(2\,M^2+7\,M\,Mp+2\,Mp^2\big)\,w^2+2\,M\,\big(-l^2\,(3\,M+2\,Mp)+Mp\,w^2\big)\,\text{Cos}[2\,x7[t]]+4\,l^2\,M\,\text{Cos}[x6[t]]^2\,\big(-2\,(M+2\,Mp)\,\text{Cos}[x7[t]]+M\,\text{Cos}[3\,x7[t]]\big)+\big(-2\,\big(l^2\,(M+2\,Mp)\,(5\,M+4\,Mp)+2\,Mp\,(2\,M+Mp)\,w^2\big)\,\text{Cos}[x7[t]]+l^2\,M\,(6\,M+8\,Mp-4\,M\,\text{Cos}[2\,x7[t]]+(5\,M+4\,Mp)\,\text{Cos}[3\,x7[t]])\big)\,\text{Sin}[2\,x6[t]]\,\text{Sin}[x7[t]]\big)\big)$$

$$
\begin{aligned}
&-(2\,w\,(8\,l^2\,(M+Mp)\,\mathrm{Cos}[x6[t]]^2\,(M-Mp\,\mathrm{Cos}[2\,x7[t]])\,\mathrm{Sin}[x6[t]]+\\
&\quad \mathrm{Cos}[x7[t]]\,((-2\,l^2\,(2\,M^2+2\,M\,Mp+Mp^2)-2\,M\,(M+2\,Mp)\,w^2+l^2\,M^2\,\mathrm{Cos}[2\,x7[t]])\,\mathrm{Sin}[x6[t]]+\\
&\quad (l^2\,(M^2+4\,M\,Mp+2\,Mp^2)-2\,M\,(M+Mp)\,w^2-4\,l^2\,(M+Mp)^2\,\mathrm{Cos}[2\,x7[t]])\,\mathrm{Sin}[3\,x6[t]])+\\
&\quad \mathrm{Cos}[x6[t]]\,(M\,(3\,l^2\,M-2\,(2\,M+Mp)\,w^2)-4\,(M+Mp)\\
&\quad ((l^2\,(M+Mp)+M\,w^2)\,\mathrm{Cos}[2\,x6[t]]+2\,l^2\,(M+Mp\,\mathrm{Cos}[2\,x6[t]])\,\mathrm{Cos}[x7[t]])-\\
&\quad l^2\,(-5\,M^2-8\,M\,Mp-4\,Mp^2+8\,(M+Mp)^2\,\mathrm{Cos}[2\,x6[t]])\,\mathrm{Cos}[2\,x7[t]]\,\mathrm{Sin}[x7[t]])\,x5'[t]+\\
&\quad l\,((\mathrm{Cos}[2\,x6[t]]\,(M\,(l^2\,(5\,M+8\,Mp)+4\,M\,w^2)-(l^2\,(11\,M^2+18\,M\,Mp+8\,Mp^2)+\\
&\quad 4\,(2\,M^2+4\,M\,Mp+Mp^2)\,w^2)\,\mathrm{Cos}[x7[t]]+\\
&\quad l^2\,(3\,M\,(3\,M+4\,Mp)\,\mathrm{Cos}[2\,x7[t]]-(M+2\,Mp)\,(5\,M+4\,Mp)\,\mathrm{Cos}[3\,x7[t]]))+\\
&\quad 2\,(l^2\,M\,(5\,M+6\,Mp)+M\,(2\,M+Mp)\,w^2-2\,(4\,l^2\,(M+Mp)^2+(2\,M^2+5\,M\,Mp+Mp^2)\,w^2)\\
&\quad \mathrm{Cos}[x7[t]]+2\,l^2\,M\,(M+2\,Mp)\,\mathrm{Cos}[2\,x7[t]]+\\
&\quad (l^2\,(7\,M^2+14\,M\,Mp+8\,Mp^2)+2\,Mp\,(2\,M+Mp)\,w^2+l^2\,(-3\,M\,(3\,M+4\,Mp)\,\mathrm{Cos}[x7[t]]+\\
&\quad (M+2\,Mp)\,(5\,M+4\,Mp)\,\mathrm{Cos}[2\,x7[t]]))\,\mathrm{Sin}[2\,x6[t]]\,\mathrm{Sin}[x7[t]]))\,x6'[t]+\\
&\quad M\,(l^2\,(6\,M+4\,Mp)+2\,(2\,M+Mp)\,w^2+M\,(l^2+4\,w^2)\,\mathrm{Cos}[2\,x6[t]]+l^2\,(-4\,M\,\mathrm{Cos}[x7[t]]+\\
&\quad (5\,M+4\,Mp)\,\mathrm{Cos}[2\,(x6[t]+x7[t])]-4\,M\,\mathrm{Cos}[2\,x6[t]+x7[t]]))\,x7'[t]))/\\
&(l\,(\mathrm{Cos}[2\,x6[t]]\,((l^2\,(13\,M^2+32\,M\,Mp+16\,Mp^2)+4\,Mp\,(2\,M+Mp)\,w^2)\,\mathrm{Cos}[2\,x7[t]]+\\
&\quad M\,(-2\,l^2\,M+4\,(M+2\,Mp)\,w^2-l^2\,(5\,M+4\,Mp)\,\mathrm{Cos}[4\,x7[t]]))+\\
&\quad 2\,(l^2\,(3\,M+2\,Mp)\,(3\,M+4\,Mp)+(2\,M^2+7\,M\,Mp+2\,Mp^2)\,w^2+2\,M\,(-l^2\,(3\,M+2\,Mp)+Mp\,w^2)\\
&\quad \mathrm{Cos}[2\,x7[t]]+4\,l^2\,M\,\mathrm{Cos}[x6[t]]^2\,(-2\,(M+2\,Mp)\,\mathrm{Cos}[x7[t]]+M\,\mathrm{Cos}[3\,x7[t]])+\\
&\quad (-2\,(l^2\,(M+2\,Mp)\,(5\,M+4\,Mp)+2\,Mp\,(2\,M+Mp)\,w^2)\,\mathrm{Cos}[x7[t]]+l^2\,M\,(6\,M+8\,Mp-4\\
&\quad M\,\mathrm{Cos}[2\,x7[t]]+(5\,M+4\,Mp)\,\mathrm{Cos}[3\,x7[t]]))\,\mathrm{Sin}[2\,x6[t]]\,\mathrm{Sin}[x7[t]])))
\end{aligned}
$$

$$\Big(w\,((l^2\,(21\,M^2+24\,M\,Mp+4\,Mp^2)+8\,Mp\,(M+Mp)\,w^2)\,\text{Sin}[x6[t]]+$$
$$8\,l^2\,M\,(M+Mp)\,\text{Sin}[3\,x6[t]]+5\,l^2\,M^2\,\text{Sin}[x6[t]-2\,x7[t]]+4\,l^2\,M\,Mp\,\text{Sin}[x6[t]-2\,x7[t]]-$$
$$24\,l^2\,M^2\,\text{Sin}[x6[t]-x7[t]]-36\,l^2\,M\,Mp\,\text{Sin}[x6[t]-x7[t]]-16\,l^2\,Mp^2\,\text{Sin}[x6[t]-x7[t]]+$$
$$4\,M^2\,w^2\,\text{Sin}[x6[t]-x7[t]]-l^2\,M^2\,\text{Sin}[3\,x6[t]-x7[t]]+l^2\,M^2\,\text{Sin}[x6[t]+x7[t]]-$$
$$8\,M^2\,w^2\,\text{Sin}[x6[t]+x7[t]]-8\,M\,Mp\,w^2\,\text{Sin}[x6[t]+x7[t]]-17\,l^2\,M^2\,\text{Sin}[3\,x6[t]+x7[t]]-$$
$$28\,l^2\,M\,Mp\,\text{Sin}[3\,x6[t]+x7[t]]-12\,l^2\,Mp^2\,\text{Sin}[3\,x6[t]+x7[t]]-$$
$$4\,M^2\,w^2\,\text{Sin}[3\,x6[t]+x7[t]]-4\,M\,Mp\,w^2\,\text{Sin}[3\,x6[t]+x7[t]]-$$
$$8\,l^2\,M^2\,\text{Sin}[x6[t]+2\,x7[t]]-4\,l^2\,M\,Mp\,\text{Sin}[x6[t]+2\,x7[t]]+8\,M^2\,w^2\,\text{Sin}[x6[t]+2\,x7[t]]-$$
$$4\,Mp^2\,w^2\,\text{Sin}[x6[t]+2\,x7[t]]+7\,l^2\,M^2\,\text{Sin}[3\,x6[t]+2\,x7[t]]+$$
$$8\,l^2\,M\,Mp\,\text{Sin}[3\,x6[t]+2\,x7[t]]+8\,M^2\,w^2\,\text{Sin}[3\,x6[t]+2\,x7[t]]+$$
$$12\,M\,Mp\,w^2\,\text{Sin}[3\,x6[t]+2\,x7[t]]+4\,Mp^2\,w^2\,\text{Sin}[3\,x6[t]+2\,x7[t]]+$$
$$5\,l^2\,M^2\,\text{Sin}[x6[t]+3\,x7[t]]+8\,l^2\,M\,Mp\,\text{Sin}[x6[t]+3\,x7[t]]+$$
$$4\,l^2\,Mp^2\,\text{Sin}[x6[t]+3\,x7[t]]+l^2\,(M+2\,Mp)\,(3\,M+2\,Mp)\,\text{Sin}[3\,x6[t]+4\,x7[t]])\,x5'[t]+$$
$$l\Big(4\Big(2\,(-(M+Mp)\,(2\,l^2\,(M+2\,Mp)-3\,Mp\,w^2+(8\,l^2\,(M+Mp)+2\,(2\,M-Mp)\,w^2)\,\text{Cos}[x7[t]]-4$$
$$l^2\,M\,\text{Cos}[2\,x7[t]])-\text{Cos}[2\,x6[t]]\,(-3\,l^2\,M\,(M+Mp)+Mp\,(5\,M+3\,Mp)$$
$$w^2+(l^2\,(M+Mp)\,(3\,M+4\,Mp)+2\,(2\,M+Mp)\,(M+2\,Mp)\,w^2)\,\text{Cos}[x7[t]]+l^2$$
$$(M+Mp)\,((M+4\,Mp)\,\text{Cos}[2\,x7[t]]+(5\,M+4\,Mp)\,\text{Cos}[3\,x7[t]]))))\,\text{Sin}\Big[\frac{x7[t]}{2}\Big]^2+$$
$$(-(l^2\,(M+Mp)\,(9\,M+4\,Mp)+2\,(2\,M+Mp)\,(M+2\,Mp)\,w^2)\,\text{Cos}[x7[t]]+$$
$$(M+Mp)\,(l^2\,(5\,M+4\,Mp)+(2\,M+Mp)\,w^2+l^2\,((9\,M+4\,Mp)\,\text{Cos}[2\,x7[t]]-$$
$$(5\,M+4\,Mp)\,\text{Cos}[3\,x7[t]]))\,\text{Sin}[2\,x6[t]]\,\text{Sin}[x7[t]]\Big)\,x6'[t]+$$
$$M\,(2\,l^2\,(5\,M+2\,Mp)+2\,(2\,M+Mp)\,w^2+M\,(5\,l^2+4\,w^2)\,\text{Cos}[2\,x6[t]]-$$
$$l^2\,M\,\text{Cos}[2\,x6[t]-x7[t]]-16\,l^2\,M\,\text{Cos}[x7[t]]-8\,l^2\,Mp\,\text{Cos}[x7[t]]-8\,M\,w^2\,\text{Cos}[x7[t]]+$$
$$4\,l^2\,M\,\text{Cos}[2\,x7[t]]+9\,l^2\,M\,\text{Cos}[2\,(x6[t]+x7[t])]+4\,l^2\,Mp\,\text{Cos}[2\,(x6[t]+x7[t])]-$$
$$10\,l^2\,M\,\text{Cos}[2\,x6[t]+x7[t]]-4\,l^2\,Mp\,\text{Cos}[2\,x6[t]+x7[t]]-8\,M\,w^2\,\text{Cos}[2\,x6[t]+x7[t]]-$$
$$4\,Mp\,w^2\,\text{Cos}[2\,x6[t]+x7[t]]-l^2\,(5\,M+4\,Mp)\,\text{Cos}[2\,x6[t]+3\,x7[t]])\,x7'[t]\Big)\Big)\Big/$$
$$(l\,(\text{Cos}[2\,x6[t]]\,((l^2\,(13\,M^2+32\,M\,Mp+16\,Mp^2)+4\,Mp\,(2\,M+Mp)\,w^2)\,\text{Cos}[2\,x7[t]]+$$
$$M\,(-2\,l^2\,M+4\,(M+2\,Mp)\,w^2-l^2\,(5\,M+4\,Mp)\,\text{Cos}[4\,x7[t]]))+$$
$$2\,(l^2\,(3\,M+2\,Mp)\,(3\,M+4\,Mp)+(2\,M^2+7\,M\,Mp+2\,Mp^2)\,w^2+2\,M\,(-l^2\,(3\,M+2\,Mp)+Mp\,w^2)$$
$$\text{Cos}[2\,x7[t]]+4\,l^2\,M\,\text{Cos}[x6[t]]^2\,(-2\,(M+2\,Mp)\,\text{Cos}[x7[t]]+M\,\text{Cos}[3\,x7[t]])+$$
$$(-2\,(l^2\,(M+2\,Mp)\,(5\,M+4\,Mp)+2\,Mp\,(2\,M+Mp)\,w^2)\,\text{Cos}[x7[t]]+l^2\,M\,(6\,M+8\,Mp-$$
$$4\,M\,\text{Cos}[2\,x7[t]]+(5\,M+4\,Mp)\,\text{Cos}[3\,x7[t]]))\,\text{Sin}[2\,x6[t]]\,\text{Sin}[x7[t]])))$$

■ Collision Guard

Note: gamma is the slope angle

```
height = First[pNSFootp[[3]] + Tan[gamma] * pNSFootp[[2]] /. {x2[t] → 0, x3[t] → 0}]
```

$$-w\,\text{Sin}[x5[t]]+2\,l\,\text{Cos}[x5[t]]\,\text{Sin}\Big[x6[t]+\frac{x7[t]}{2}\Big]\,\text{Sin}\Big[\frac{x7[t]}{2}\Big]+$$
$$l\,(-\text{Sin}[x6[t]]+\text{Sin}[x6[t]+x7[t]])\,\text{Tan}[\text{gamma}]$$

```
Avect = {{D[height, x5[t]], D[height, x6[t]], D[height, x7[t]]}};
holonomicTraj = First[First[Avect.{{x5'[t]}, {x6'[t]}, {x7'[t]}}]] // FullSimplify
```

$$-\left(w \cos[x5[t]] + 2\, l \sin[x5[t]] \sin\left[x6[t] + \frac{x7[t]}{2}\right] \sin\left[\frac{x7[t]}{2}\right]\right) x5'[t] +$$
$$l \left(2 \cos[x5[t]] \cos\left[x6[t] + \frac{x7[t]}{2}\right] \sin\left[\frac{x7[t]}{2}\right] +$$
$$(-\cos[x6[t]] + \cos[x6[t] + x7[t]]) \tan[\text{gamma}]\right) x6'[t] +$$
$$l\, (\cos[x5[t]] \sin[x6[t] + x7[t]] + \cos[x6[t] + x7[t]] \tan[\text{gamma}])\, x7'[t]$$

The guard is the zero-level set of the height function:

```
height == 0
```

$$-w \sin[x5[t]] + 2\, l \cos[x5[t]] \sin\left[x6[t] + \frac{x7[t]}{2}\right] \sin\left[\frac{x7[t]}{2}\right] +$$
$$l\, (-\sin[x6[t]] + \sin[x6[t] + x7[t]]) \tan[\text{gamma}] == 0$$

and the negative region of the holonomic constraint's trajectory:

```
holonomicTraj < 0
```

$$-\left(w \cos[x5[t]] + 2\, l \sin[x5[t]] \sin\left[x6[t] + \frac{x7[t]}{2}\right] \sin\left[\frac{x7[t]}{2}\right]\right) x5'[t] +$$
$$l \left(2 \cos[x5[t]] \cos\left[x6[t] + \frac{x7[t]}{2}\right] \sin\left[\frac{x7[t]}{2}\right] +$$
$$(-\cos[x6[t]] + \cos[x6[t] + x7[t]]) \tan[\text{gamma}]\right) x6'[t] +$$
$$l\, (\cos[x5[t]] \sin[x6[t] + x7[t]] + \cos[x6[t] + x7[t]] \tan[\text{gamma}])\, x7'[t] < 0$$