Mathematical Model for 3D Point-footed, Midleg-Mass, Hipped Walker with Yaw

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```
<< H:\Screws.m
<< H:\RobotLinks.m
```

Variables

```
x4 --> stance yaw angle
x5 --> stance roll angle
x6 --> stance pitch angle
x7 --> swing pitch angle
```

Constants

```
w --> hip width
l --> leg length
Mp --> pelvis / hip mass
M --> midleg mass
gamma --> slope angle
```

3D Walker Open Chain Kinematics

Foot/Leg links

```
q = {{x1[t]}, {x2[t]}, {x3[t]}, {x4[t]}, {x5[t]}, {x6[t]}, {x7[t]}};
qdot = D[q, t];
```

```
ax1 = {1, 0, 0, 0, 0, 0};
ax2 = {0, 1, 0, 0, 0, 0};
ax3 = {0, 0, 1, 0, 0, 0};
ax4 = {0, 0, 0, 0, 0, 1};
ax5 = {0, 0, 0, 0, 1, 0};
ax6 = {0, 0, 0, 1, 0, 0};
ax7 = Flatten[Append[Cross[{0, 0, l}, {1, 0, 0}], {1, 0, 0}]];

gst01 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, l / 2}, {0, 0, 0, 1}};
gst02 = {{1, 0, 0, w / 2}, {0, 1, 0, 0}, {0, 0, 1, l}, {0, 0, 0, 1}};
gst03 = {{1, 0, 0, w}, {0, 1, 0, 0}, {0, 0, 1, l / 2}, {0, 0, 0, 1}};

J1 = BodyJacobian[{ax1, x1[t]}, {ax2, x2[t]}, {ax3, x3[t]}, {ax4, x4[t]},
    {ax5, x5[t]}, {ax6, x6[t]}, {{0, 0, 0, 0, 0, 0}, x7[t]}, gst01] // FullSimplify;
J2 = BodyJacobian[{ax1, x1[t]}, {ax2, x2[t]}, {ax3, x3[t]}, {ax4, x4[t]},
    {ax5, x5[t]}, {ax6, x6[t]}, {{0, 0, 0, 0, 0, 0}, x7[t]}, gst02] // FullSimplify;
J3 = BodyJacobian[{ax1, x1[t]}, {ax2, x2[t]}, {ax3, x3[t]}, {ax4, x4[t]},
    {ax5, x5[t]}, {ax6, x6[t]}, {ax7, x7[t]}, gst03] // FullSimplify;
MatrixForm[J1]
MatrixForm[J2]
MatrixForm[J3]
```

```
( Cos[x4[t]] Cos[x5[t]]                                          Cos[x5[t]] Sin[x4[t]]
  -Cos[x6[t]] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t]]      Cos[x4[t]] Cos[x6[t]] + Sin[x4[
  Cos[x4[t]] Cos[x6[t]] Sin[x5[t]] + Sin[x4[t]] Sin[x6[t]]       Cos[x6[t]] Sin[x4[t]] Sin[x5[t
  0                                                              0
  0                                                              0
  0                                                              0

( Cos[x4[t]] Cos[x5[t]]                                          Cos[x5[t]] Sin[x4[t]]
  -Cos[x6[t]] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t]]      Cos[x4[t]] Cos[x6[t]] + Sin[x4[
  Cos[x4[t]] Cos[x6[t]] Sin[x5[t]] + Sin[x4[t]] Sin[x6[t]]       Cos[x6[t]] Sin[x4[t]] Sin[x5[t
  0                                                              0
  0                                                              0
  0                                                              0

( Cos[x4[t]] Cos[x5[t]]                                                              Cos[x5[t]] Sin[x
  -Cos[x6[t] + x7[t]] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]]          Cos[x4[t]] Cos[x
  Cos[x4[t]] Cos[x6[t] + x7[t]] Sin[x5[t]] + Sin[x4[t]] Sin[x6[t] + x7[t]]           Cos[x6[t] + x7[t]
  0                                                                                  0
  0                                                                                  0
  0                                                                                  0
```

```
genM1 = M * {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0},
    {0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}};
genM2 = Mp * {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
    {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}};
genM3 = M * {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
    {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}};
```

```
MatrixForm[Mmatrix = Transpose[J1].genM1.J1 +
    Transpose[J2].genM2.J2 + Transpose[J3].genM3.J3 // FullSimplify]
```

```
2 M + Mp
0
0
1/2 (-(2 M + Mp) w Cos[x5[t]] Sin[x4[t]] + l (-(3 M + 2 Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin
1/2 Cos[x4[t]] (l Cos[x5[t]] ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) - (2 M + Mp) w Sin[x5
1/2 (l ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) Sin[x4[t]] + l Cos[x4[t]] Sin[x5[t]] (-(3
1/2 l M (-Cos[x6[t] + x7[t]] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]])
```

```
MatrixForm[Mmatrix /. w → 0 // FullSimplify]
```

```
2 M + Mp
0
0
1/2 (l (-(3 M + 2 Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin[x4[t]] Sin[x5[t]] + l Cos[x4[t]] ((3
1/2 l Cos[x4[t]] Cos[x5[t]] ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]])
1/2 (l ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) Sin[x4[t]] + l Cos[x4[t]] Sin[x5[t]] (-(3
1/2 l M (-Cos[x6[t] + x7[t]] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]])
```

```
KE = First[First[1 / 2 Transpose[qdot].Mmatrix.qdot]] // Simplify;

KE /. {x1[t] → 0, x2[t] → 0, x3[t] → 0, x1'[t] → 0, x2'[t] → 0, x3'[t] → 0} // FullSimplify
```

```
 1
16
 ((Cos[x5[t]]^2 (l^2 (6 M + 4 Mp) + 2 (4 M + Mp) w^2 - l^2 ((5 M + 4 Mp) Cos[2 x6[t]] + M (4 Cos[x7[t]] +
                  Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]])))) +
       4 l (l (3 M + 2 Mp - 2 M Cos[x7[t]]) Sin[x5[t]]^2 + w ((2 M + Mp) Cos[x6[t]] -
             M Cos[x6[t] + x7[t]]) Sin[2 x5[t]])) x4'[t]^2 +
    (l^2 (6 M + 4 Mp) + 2 (4 M + Mp) w^2 + l^2 ((5 M + 4 Mp) Cos[2 x6[t]] +
          M (-4 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]])))) x5'[t]^2 +
    2 l x4'[t] ((4 w Sin[x5[t]] (-(2 M + Mp) Sin[x6[t]] + M Sin[x6[t] + x7[t]]) + l Cos[x5[t]]
            ((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]])))
         x5'[t] - 4 (w Cos[x5[t]] ((2 M + Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) +
           l (3 M + 2 Mp - 2 M Cos[x7[t]]) Sin[x5[t]]) x6'[t] +
       2 M (2 w Cos[x5[t]] Cos[x6[t] + x7[t]] + l (-1 + 2 Cos[x7[t]]) Sin[x5[t]]) x7'[t]) +
    8 l w x5'[t] (((2 M + Mp) Sin[x6[t]] - M Sin[x6[t] + x7[t]]) x6'[t] -
       M Sin[x6[t] + x7[t]] x7'[t]) +
    2 l^2 ((6 M + 4 Mp - 4 M Cos[x7[t]]) x6'[t]^2 + 2 M (1 - 2 Cos[x7[t]]) x6'[t] x7'[t] + M x7'[t]^2))
```

■ Potential Term

```
MatrixForm[e1 = TwistExp[ax1, x1[t]]]
MatrixForm[e2 = TwistExp[ax2, x2[t]]]
MatrixForm[e3 = TwistExp[ax3, x3[t]]]
MatrixForm[e4 = TwistExp[ax4, x4[t]]]
MatrixForm[e5 = TwistExp[ax5, x5[t]]]
MatrixForm[e6 = TwistExp[ax6, x6[t]]]
MatrixForm[e7 = TwistExp[ax7, x7[t]]]
```

$$\begin{pmatrix} 1 & 0 & 0 & x1[t] \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & x2[t] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & x3[t] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} Cos[x4[t]] & -Sin[x4[t]] & 0 & 0 \\ Sin[x4[t]] & Cos[x4[t]] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} Cos[x5[t]] & 0 & Sin[x5[t]] & 0 \\ 0 & 1 & 0 & 0 \\ -Sin[x5[t]] & 0 & Cos[x5[t]] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos[x6[t]] & -Sin[x6[t]] & 0 \\ 0 & Sin[x6[t]] & Cos[x6[t]] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos[x7[t]] & -Sin[x7[t]] & l\,Sin[x7[t]] \\ 0 & Sin[x7[t]] & Cos[x7[t]] & l\,(1 - Cos[x7[t]]) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
pS = {{0}, {0}, {l / 2}, {1}};
pH = {{w / 2}, {0}, {l}, {1}};
pNS = {{w}, {0}, {l / 2}, {1}};
```

```
MatrixForm[pSp = e1.e2.e3.e4.e5.e6.pS // FullSimplify]
MatrixForm[pHp = e1.e2.e3.e4.e5.e6.pH // FullSimplify]
MatrixForm[pNSp = e1.e2.e3.e4.e5.e6.e7.pNS // FullSimplify]
```

$$\begin{pmatrix} \frac{1}{2} l (\text{Cos}[x4[t]] \text{Cos}[x6[t]] \text{Sin}[x5[t]] + \text{Sin}[x4[t]] \text{Sin}[x6[t]]) + x1[t] \\ \frac{1}{2} l (\text{Cos}[x6[t]] \text{Sin}[x4[t]] \text{Sin}[x5[t]] - \text{Cos}[x4[t]] \text{Sin}[x6[t]]) + x2[t] \\ \frac{1}{2} l \text{Cos}[x5[t]] \text{Cos}[x6[t]] + x3[t] \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[x4[t]] (\frac{1}{2} w \text{Cos}[x5[t]] + l \text{Cos}[x6[t]] \text{Sin}[x5[t]]) + l \text{Sin}[x4[t]] \text{Sin}[x6[t]] + x1[t] \\ \frac{1}{2} w \text{Cos}[x5[t]] \text{Sin}[x4[t]] + l \text{Cos}[x6[t]] \text{Sin}[x4[t]] \text{Sin}[x5[t]] - l \text{Cos}[x4[t]] \text{Sin}[x6[t]] + x2 \\ l \text{Cos}[x5[t]] \text{Cos}[x6[t]] - \frac{1}{2} w \text{Sin}[x5[t]] + x3[t] \\ 1 \end{pmatrix}$$

```
General::spell :
 Possible spelling error: new symbol name "pNSp" is similar to existing symbols {pNS, pSp}. More…
```

$$\begin{pmatrix} \frac{1}{2} (\text{Cos}[x4[t]] (2 w \text{Cos}[x5[t]] - l (-2 \text{Cos}[x6[t]] + \text{Cos}[x6[t] + x7[t]]) \text{Sin}[x5[t]]) - l \text{Sin}[x4[\\ \frac{1}{2} (2 w \text{Cos}[x5[t]] \text{Sin}[x4[t]] - l (-2 \text{Cos}[x6[t]] + \text{Cos}[x6[t] + x7[t]]) \text{Sin}[x4[t]] \text{Sin}[x5[t]] + \\ -\frac{1}{2} l \text{Cos}[x5[t]] (-2 \text{Cos}[x6[t]] + \text{Cos}[x6[t] + x7[t]]) - w \text{Sin}[x5[t]] + x3[t] \\ 1 \end{pmatrix}$$

```
PE = First[M * g * pSp[[3]] + Mp * g * pHp[[3]] + M * g * pNSp[[3]]] // FullSimplify
```

$$\frac{1}{2} g (l \text{Cos}[x5[t]] ((3 M + 2 Mp) \text{Cos}[x6[t]] - M \text{Cos}[x6[t] + x7[t]]) - (2 M + Mp) w \text{Sin}[x5[t]] + 2 (2 M + Mp) x3[t])$$

```
% /. w → 0
```

$$\frac{1}{2} g (l \text{Cos}[x5[t]] ((3 M + 2 Mp) \text{Cos}[x6[t]] - M \text{Cos}[x6[t] + x7[t]]) + 2 (2 M + Mp) x3[t])$$

3D Walker Dynamics

```
Lagrangian =
  KE - PE /. {x1[t] → 0, x2[t] → 0, x3[t] → 0, x1'[t] → 0, x2'[t] → 0, x3'[t] → 0};

eq1 = D[D[Lagrangian, x4'[t]], t] - D[Lagrangian, x4[t]] // FullSimplify
eq2 = D[D[Lagrangian, x5'[t]], t] - D[Lagrangian, x5[t]] // FullSimplify
eq3 = D[D[Lagrangian, x6'[t]], t] - D[Lagrangian, x6[t]] // FullSimplify
eq4 = D[D[Lagrangian, x7'[t]], t] - D[Lagrangian, x7[t]] // FullSimplify
```

```
1/8 (-l (4 w Cos[x5[t]] ((2 M + Mp) Sin[x6[t]] - M Sin[x6[t] + x7[t]]) + l Sin[x5[t]]
      ((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]]))) x5'[t]^2 +
  8 l M w Cos[x5[t]] Sin[x6[t]] x6'[t]^2 + 4 l Mp w Cos[x5[t]] Sin[x6[t]] x6'[t]^2 -
  4 l M w Cos[x5[t]] Sin[x6[t] + x7[t]] x6'[t]^2 - 8 l^2 M Sin[x5[t]] Sin[x7[t]] x6'[t] x7'[t] -
  8 l M w Cos[x5[t]] Sin[x6[t] + x7[t]] x6'[t] x7'[t] -
  4 l^2 M Sin[x5[t]] Sin[x7[t]] x7'[t]^2 - 4 l M w Cos[x5[t]] Sin[x6[t] + x7[t]] x7'[t]^2 +
  2 l^2 Cos[x5[t]] x5'[t] ((-6 M - 4 Mp + (5 M + 4 Mp) Cos[2 x6[t]] +
        M (4 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]])) x6'[t] +
     M (-1 + 2 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 2 Cos[2 x6[t] + x7[t]]) x7'[t]) +
  x4'[t] ((8 l w Cos[2 x5[t]] ((2 M + Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) +
        (l^2 (6 M + 4 Mp) - 2 (4 M + Mp) w^2 + l^2 ((5 M + 4 Mp) Cos[2 x6[t]] + M (-4 Cos[x7[t]] +
                 Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]]))) Sin[2 x5[t]]) x5'[t] +
     2 l ((2 w Sin[2 x5[t]] (-(2 M + Mp) Sin[x6[t]] + M Sin[x6[t] + x7[t]]) + l Cos[x5[t]]^2
            ((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]])))
          x6'[t] + M (4 l Sin[x5[t]]^2 Sin[x7[t]] + 2 w Sin[2 x5[t]] Sin[x6[t] + x7[t]] +
             l Cos[x5[t]]^2 (2 Sin[x7[t]] + Sin[2 (x6[t] + x7[t])] - 2 Sin[2 x6[t] + x7[t]]))
          x7'[t])) + 6 l^2 M Cos[x5[t]]^2 x4''[t] + 4 l^2 Mp Cos[x5[t]]^2 x4''[t] +
  8 M w^2 Cos[x5[t]]^2 x4''[t] + 2 Mp w^2 Cos[x5[t]]^2 x4''[t] - 5 l^2 M Cos[x5[t]]^2
   Cos[2 x6[t]] x4''[t] -
  4 l^2 Mp Cos[x5[t]]^2 Cos[2 x6[t]] x4''[t] -
  4 l^2 M Cos[x5[t]]^2 Cos[x7[t]] x4''[t] -
  l^2 M Cos[x5[t]]^2
   Cos[2 (x6[t] + x7[t])] x4''[t] +
  4 l^2 M Cos[x5[t]]^2 Cos[2 x6[t] + x7[t]] x4''[t] +
  12 l^2 M Sin[x5[t]]^2 x4''[t] +
  8 l^2 Mp Sin[x5[t]]^2 x4''[t] -
  8 l^2 M Cos[x7[t]] Sin[x5[t]]^2 x4''[t] +
  8 l M w Cos[x6[t]] Sin[2 x5[t]] x4''[t] +
  4 l Mp w Cos[x6[t]] Sin[2 x5[t]] x4''[t] -
  4 l M w Cos[x6[t] + x7[t]] Sin[2 x5[t]] x4''[t] -
  8 l M w Sin[x5[t]] Sin[x6[t]] x5''[t] -
  4 l Mp w Sin[x5[t]] Sin[x6[t]] x5''[t] +
  5 l^2 M Cos[x5[t]] Sin[2 x6[t]] x5''[t] +
  4 l^2 Mp Cos[x5[t]] Sin[2 x6[t]] x5''[t] +
  4 l M w Sin[x5[t]] Sin[x6[t] + x7[t]] x5''[t] +
  l^2 M Cos[x5[t]] Sin[2 (x6[t] + x7[t])] x5''[t] -
  4 l^2 M Cos[x5[t]] Sin[2 x6[t] + x7[t]] x5''[t] -
  8 l M w Cos[x5[t]] Cos[x6[t]] x6''[t] -
  4 l Mp w Cos[x5[t]] Cos[x6[t]] x6''[t] +
  4 l M w Cos[x5[t]] Cos[x6[t] + x7[t]] x6''[t] -
  12 l^2 M Sin[x5[t]] x6''[t] - 8 l^2 Mp Sin[x5[t]] x6''[t] +
  8 l^2 M Cos[x7[t]] Sin[x5[t]] x6''[t] +
  2 l M (2 w Cos[x5[t]] Cos[x6[t] + x7[t]] + l (-1 + 2 Cos[x7[t]]) Sin[x5[t]]) x7''[t])
```

```
1/16 (-8 g (2 M + Mp) w Cos[x5[t]] +
   8 g l (-(3 M + 2 Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin[x5[t]] -
   (8 l w Cos[2 x5[t]] ((2 M + Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) +
      (l^2 (6 M + 4 Mp) - 2 (4 M + Mp) w^2 + l^2 ((5 M + 4 Mp) Cos[2 x6[t]] +
            M (-4 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]]))) Sin[2 x5[t]])
    x4′[t]^2 + 4 l x4′[t] ((l Cos[x5[t]] (6 M + 4 Mp + (5 M + 4 Mp) Cos[2 x6[t]] +
            M (-4 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]])) +
         4 w (-(2 M + Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin[x5[t]]) x6′[t] +
      M (l Cos[x5[t]] (1 - 2 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 2 Cos[2 x6[t] + x7[t]]) +
         4 w Cos[x6[t] + x7[t]] Sin[x5[t]]) x7′[t]) +
   2 (4 l w ((2 M + Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) x6′[t]^2 -
      8 l M w Cos[x6[t] + x7[t]] x6′[t] x7′[t] - 4 l M w Cos[x6[t] + x7[t]] x7′[t]^2 + 2 l^2 x5′[t]
       (-((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]]))
          x6′[t] + M (2 Sin[x7[t]] - Sin[2 (x6[t] + x7[t])] + 2 Sin[2 x6[t] + x7[t]]) x7′[t]) +
      4 l w Sin[x5[t]] (-(2 M + Mp) Sin[x6[t]] + M Sin[x6[t] + x7[t]]) x4″[t] + l^2 Cos[x5[t]]
       ((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]])) x4″[t] +
      6 l^2 M x5″[t] + 4 l^2 Mp x5″[t] + 8 M w^2 x5″[t] + 2 Mp w^2 x5″[t] +
      5 l^2 M Cos[2 x6[t]] x5″[t] + 4 l^2 Mp Cos[2 x6[t]] x5″[t] - 4 l^2 M Cos[x7[t]] x5″[t] +
      l^2 M Cos[2 (x6[t] + x7[t])] x5″[t] - 4 l^2 M Cos[2 x6[t] + x7[t]] x5″[t] +
      8 l M w Sin[x6[t]] x6″[t] + 4 l Mp w Sin[x6[t]] x6″[t] -
      4 l M w Sin[x6[t] + x7[t]] x6″[t] - 4 l M w Sin[x6[t] + x7[t]] x7″[t]))

-1/8 l ((2 w Sin[2 x5[t]] (-(2 M + Mp) Sin[x6[t]] + M Sin[x6[t] + x7[t]]) + l Cos[x5[t]]^2
       ((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]]))) x4′[t]^2 -
   l ((5 M + 4 Mp) Sin[2 x6[t]] + M (Sin[2 (x6[t] + x7[t])] - 4 Sin[2 x6[t] + x7[t]])) x5′[t]^2 +
   2 x4′[t] ((l Cos[x5[t]] (6 M + 4 Mp + (5 M + 4 Mp) Cos[2 x6[t]] +
            M (-4 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 4 Cos[2 x6[t] + x7[t]])) +
         4 w (-(2 M + Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin[x5[t]])
       x5′[t] + 4 l M Sin[x5[t]] Sin[x7[t]] x7′[t]) +
   4 (g Cos[x5[t]] ((3 M + 2 Mp) Sin[x6[t]] - M Sin[x6[t] + x7[t]]) -
      2 l M Sin[x7[t]] x6′[t] x7′[t] - l M Sin[x7[t]] x7′[t]^2 +
      2 M w Cos[x5[t]] Cos[x6[t]] x4″[t] + Mp w Cos[x5[t]] Cos[x6[t]] x4″[t] -
      M w Cos[x5[t]] Cos[x6[t] + x7[t]] x4″[t] + 3 l M Sin[x5[t]] x4″[t] +
      2 l Mp Sin[x5[t]] x4″[t] - 2 l M Cos[x7[t]] Sin[x5[t]] x4″[t] -
      2 M w Sin[x6[t]] x5″[t] - Mp w Sin[x6[t]] x5″[t] + M w Sin[x6[t] + x7[t]] x5″[t] +
      l (-3 M - 2 Mp + 2 M Cos[x7[t]]) x6″[t]) + 2 l M (-1 + 2 Cos[x7[t]]) x7″[t])

-1/8 l M ((4 l Sin[x5[t]]^2 Sin[x7[t]] + 2 w Sin[2 x5[t]] Sin[x6[t] + x7[t]] +
      l Cos[x5[t]]^2 (2 Sin[x7[t]] + Sin[2 (x6[t] + x7[t])] - 2 Sin[2 x6[t] + x7[t]])) x4′[t]^2 +
   l (2 Sin[x7[t]] - Sin[2 (x6[t] + x7[t])] + 2 Sin[2 x6[t] + x7[t]]) x5′[t]^2 +
   2 x4′[t] ((l Cos[x5[t]] (1 - 2 Cos[x7[t]] + Cos[2 (x6[t] + x7[t])] - 2 Cos[2 x6[t] + x7[t]]) +
         4 w Cos[x6[t] + x7[t]] Sin[x5[t]]) x5′[t] - 4 l Sin[x5[t]] Sin[x7[t]] x6′[t]) -
   2 (2 w Cos[x5[t]] Cos[x6[t] + x7[t]] x4″[t] + 2 Sin[x6[t] + x7[t]] (g Cos[x5[t]] - w x5″[t]) +
      l (-2 Sin[x7[t]] x6′[t]^2 + (-1 + 2 Cos[x7[t]]) (Sin[x5[t]] x4″[t] - x6″[t]) + x7″[t])))
```

```
Solve[{eq1 == 0 , eq2 == 0, eq3 == 0, eq4 == 0}, {x4''[t], x5''[t], x6''[t], x7''[t]}];
```

Without a hip:

```
Solve[{eq1 == 0 /. w → 0, eq2 == 0 /. w → 0, eq3 == 0 /. w → 0, eq4 == 0 /. w → 0},
  {x4''[t], x5''[t], x6''[t], x7''[t]}] // FullSimplify
```

```
{{x5″[t] →
    1/(l (M + 4 Mp)) (Csc[x7[t]] (2 g Sin[x5[t]] (-M Sin[x6[t]] + (M + 2 Mp) Sin[x6[t] + x7[t]]) +
        l (M + 4 Mp) (Cos[x5[t]] Sin[x7[t]] x4′[t] (Sin[x5[t]] x4′[t] - 2 x6′[t]) + 2 Sin[x6[t]]
            (Cos[x5[t]] Cos[x6[t] + x7[t]] x4′[t] - Sin[x6[t] + x7[t]] x5′[t]) x7′[t]))), x4″[
   t] → 1/(l (M + 4 Mp)) (2 Csc[x7[t]] Sec[x5[t]] (g (M Cos[x6[t]] - (M + 2 Mp) Cos[x6[t] + x7[t]])
         Sin[x5[t]] + l (M + 4 Mp) (Sin[x7[t]] x5′[t] x6′[t] - Cos[x6[t]]
            (Cos[x5[t]] Cos[x6[t] + x7[t]] x4′[t] - Sin[x6[t] + x7[t]] x5′[t]) x7′[t]))),
  x6″[t] → -(Csc[x7[t]] Sec[x5[t]] (16 g (Cos[x6[t]] ((-5 M^2 - 20 M Mp - 16 Mp^2 + (3 M^2 + 12 M Mp +
                      16 Mp^2) Cos[2 x5[t]]) Cos[x7[t]] - M (-3 M - 8 Mp + M Cos[2 x5[t]])
                 Cos[3 x7[t]] + 4 M (3 M + 4 Mp - 2 M Cos[2 x7[t]]) Sin[x5[t]]^2) +
              Sin[x6[t]] ((13 M^2 + 48 M Mp + 32 Mp^2 - 3 M^2 Cos[2 x5[t]]) Sin[x7[t]] + M
                 (-3 M - 8 Mp + M Cos[2 x5[t]]) Sin[3 x7[t]])) + 8 l (M + 4 Mp)
           (Sin[x7[t]] (M Cos[3 x5[t]] (Sin[x7[t]] - Sin[2 x7[t]]) + 5 M Cos[x5[t]]
                 (-Sin[x7[t]] + Sin[2 x7[t]]) + Cos[x5[t]]^3 ((6 M + 8 Mp) Sin[2 x6[t]] +
                   4 M (Cos[2 (x6[t] + x7[t])] Sin[x7[t]] - Sin[2 (x6[t] + x7[t])]))) x4′[t]^2 -
              2 Cos[x5[t]] Sin[x7[t]] ((3 M + 4 Mp) Sin[2 x6[t]] + 4 M (Cos[x6[t] + x7[t]]^2
                    Sin[x7[t]] - Cos[x6[t]] Sin[x6[t] + 2 x7[t]])) x5′[t]^2 + 4 x4′[t] (Sin[
                  x7[t]] (Cos[x5[t]]^2 (3 M + 4 Mp + (3 M + 4 Mp) Cos[2 x6[t]] - 2 M (Cos[2 x7[t]] +
                          Cos[2 (x6[t] + x7[t])] + Sin[x7[t]] Sin[2 (x6[t] + x7[t])]))) x5′[t] +
                   2 M (1 - 2 Cos[x7[t]]) Sin[2 x5[t]] Sin[x7[t]] x6′[t]) + 1/2
                 Sin[2 x5[t]] (2 Cos[x6[t]]^2 Cos[x7[t]] (-3 M - 4 Mp + 2 M Cos[2 x7[t]]) +
                    Sin[x7[t]] ((3 M + 4 Mp - 2 M Cos[2 x7[t]]) Sin[2 x6[t]] + 4 M Sin[x7[t]]))
                 x7′[t]) - 8 (-3 M - 4 Mp + 2 M Cos[2 x7[t]]) Sin[x5[t]] x5′[t]
               (Sin[x7[t]] x6′[t] + Cos[x6[t]] Sin[x6[t] + x7[t]] x7′[t]) +
              8 M Cos[x5[t]] Sin[x7[t]]^2 (2 Cos[x7[t]] x6′[t]^2 - (x6′[t] + x7′[t])^2)))) /
     (32 l (M + 4 Mp) (-3 M - 4 Mp + 2 M Cos[2 x7[t]]))), x7″[
   t] →
    1/(l (-3 M - 4 Mp + 2 M Cos[2 x7[t]]))
    (2
       g
       Cos[x5[t]]
       (-(3 M + 2 Mp) (-1 + 2 Cos[x7[t]]) Sin[x6[t]] +
         (5 M + 4 Mp - 2 M Cos[x7[t]]) Sin[x6[t] + x7[t]]) +
      l (-9 M - 6 Mp + 6 M Cos[x7[t]] + Cos[2 x5[t]] (3 M + 2 Mp - 2 M Cos[x7[t]]) +
          Cos[x5[t]]^2 (M Cos[2 (x6[t] + x7[t])] +
              (5 M + 4 Mp) (Cos[2 x6[t]] - Cos[2 x6[t] + x7[t]]))) Sin[x7[t]] x4′[t]^2 +
      l Sin[x7[t]] (-(6 M + 4 Mp + (5 M + 4 Mp) Cos[2 x6[t]] - 4 M Cos[x7[t]] +
              M Cos[2 (x6[t] + x7[t])] - (5 M + 4 Mp) Cos[2 x6[t] + x7[t]]) x5′[t]^2 +
         4 (-3 M - 2 Mp + 2 M Cos[x7[t]]) x6′[t]^2 + 4 M (-1 + 2 Cos[x7[t]]) x6′[t] x7′[t] +
         2 M (-1 + 2 Cos[x7[t]]) x7′[t]^2) +
      2 l Sin[x7[t]] x4′[t] (-Cos[x5[t]] (M Sin[2 (x6[t] + x7[t])] +
              (5 M + 4 Mp) (Sin[2 x6[t]] - Sin[2 x6[t] + x7[t]])) x5′[t] +
         2 Sin[x5[t]] ((6 M + 4 Mp - 4 M Cos[x7[t]]) x6′[t] + M (1 - 2 Cos[x7[t]]) x7′[t])))}}
```

Impact Equations

```
pNSFoot = {{w}, {0}, {0}, {1}};
MatrixForm[pNSFootp = e1.e2.e3.e4.e5.e6.e7.pNSFoot // FullSimplify]
```

```
General::spell1 :
 Possible spelling error: new symbol name "pNSFootp" is similar to existing symbol "pNSFoot". More…
```

```
( w Cos[x4[t]] Cos[x5[t]] - 2 l (Cos[x6[t] + x7[t]/2] Sin[x4[t]] - Cos[x4[t]] Sin[x5[t]] Sin[x6[t
  w Cos[x5[t]] Sin[x4[t]] + 2 l (Cos[x4[t]] Cos[x6[t] + x7[t]/2] + Sin[x4[t]] Sin[x5[t]] Sin[x6[t
  -w Sin[x5[t]] + 2 l Cos[x5[t]] Sin[x6[t] + x7[t]/2] Sin[x7[t]/2] + x3[t]
  1
```

```
Pos1 = First[pNSFootp[[1]]];
Pos2 = First[pNSFootp[[2]]];
Pos3 = First[pNSFootp[[3]]];
MatrixForm[
 F = FullSimplify[{{D[Pos1, x1[t]], D[Pos1, x2[t]], D[Pos1, x3[t]], D[Pos1, x4[t]],
     D[Pos1, x5[t]], D[Pos1, x6[t]], D[Pos1, x7[t]]},
    {D[Pos2, x1[t]], D[Pos2, x2[t]], D[Pos2, x3[t]], D[Pos2, x4[t]],
     D[Pos2, x5[t]], D[Pos2, x6[t]], D[Pos2, x7[t]]},
    {D[Pos3, x1[t]], D[Pos3, x2[t]], D[Pos3, x3[t]], D[Pos3, x4[t]],
     D[Pos3, x5[t]], D[Pos3, x6[t]], D[Pos3, x7[t]]}}]]
```

```
( 1  0  0  -w Cos[x5[t]] Sin[x4[t]] - 2 l (Cos[x4[t]] Cos[x6[t] + x7[t]/2] + Sin[x4[t]] Sin[x5[t]
  0  1  0  w Cos[x4[t]] Cos[x5[t]] + 2 l (-Cos[x6[t] + x7[t]/2] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]
  0  0  1  0
```

```
MatrixForm[nFT = Join[-Transpose[F], {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}]]
```

```
( -1
  0
  0
  w Cos[x5[t]] Sin[x4[t]] + 2 l (Cos[x4[t]] Cos[x6[t] + x7[t]/2] + Sin[x4[t]] Sin[x5[t]] Sin[x6[t
  -Cos[x4[t]] (-w Sin[x5[t]] + 2 l Cos[x5[t]] Sin[x6[t] + x7[t]/2] Sin[x7[t]/2])
  -2 l (Cos[x4[t]] Cos[x6[t] + x7[t]/2] Sin[x5[t]] + Sin[x4[t]] Sin[x6[t] + x7[t]/2]) Sin[x7[t]/2]
  l Cos[x6[t] + x7[t]] Sin[x4[t]] - l Cos[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]]
  0
  0
  0
```

```
MatrixForm[FullMatrix = Transpose[Join[Transpose[Join[Mmatrix, F]], Transpose[nFT]]]]
```

```
2 M + Mp
0
0
1/2 (-(2 M + Mp) w Cos[x5[t]] Sin[x4[t]] + l (-(3 M + 2 Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin
1/2 Cos[x4[t]] (l Cos[x5[t]] ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) - (2 M + Mp) w Sin[x5
1/2 (l ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) Sin[x4[t]] + l Cos[x4[t]] Sin[x5[t]] (-(3
1/2 l M (-Cos[x6[t] + x7[t]] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]])
1
0
0
```

```
MatrixForm[FullMatrix /. w → 0 // FullSimplify];
```

```
MatrixForm[DMatrix = Join[Mmatrix.{0, 0, 0, x4'[t], x5'[t], x6'[t], x7'[t]}, {0, 0, 0}]]
```

```
1/2 (-(2 M + Mp) w Cos[x5[t]] Sin[x4[t]] + l (-(3 M + 2 Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin
1/2 (Cos[x4[t]] ((2 M + Mp) w Cos[x5[t]] + l ((3 M + 2 Mp) Cos[x6[t]] - M Cos[x6[t] + x7[t]]) Sin[x
1/2 (-(2 M + Mp) w Cos[x5[t]] + l (-(3 M + 2 Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) Sin[x5[t]]) x5'
1/8 (Cos[x5[t]]^2 (l^2 (6 M + 4 Mp) + 2 (4 M + Mp) w^2 - l^2 ((5 M + 4 Mp) Cos[2 x6[t]] + M (4 Cos[x7[t]] +
1/8 l (4 w Sin[x5[t]] (-(2 M + Mp) Sin[x6[t]] + M Sin[x6[t] + x7[t]]) + l Cos[x5[t]] ((5 M + 4 Mp) S
1/2 l (w Cos[x5[t]] (-(2 M + Mp) Cos[x6[t]] + M Cos[x6[t] + x7[t]]) + l (-3 M - 2 Mp + 2 M Cos[x7[t]
1/4 l M (2 w Cos[x5[t]] Cos[x6[t] + x7[t]] + l (-1 + 2 Cos[x7[t]]) Sin[x5[t]]) x4'[t] - 1/2 l M w Sin
0
0
0
```

First, we have the angular positions for 4,5,6,7:

```
{x4impact = π + x4[t], x5impact = x5[t], x6impact = x6[t] + x7[t], x7impact = -x7[t]};
```

We are interested in the angular velocities for 4,5,6,7 (as of now these are incomputable for the hipped walker):

```
K = Inverse[FullMatrix].DMatrix;
```

```
x4dotimpact = FullSimplify[K[[4]]]
x5dotimpact = FullSimplify[K[[5]]]
x6dotimpact = FullSimplify[K[[6]] + K[[7]]]
x7dotimpact = -FullSimplify[K[[7]]]
```

```
$Aborted
```

```
$Aborted
```

```
$Aborted
```

```
$Aborted
```

■ Without Hip

```
FullMatrix = FullMatrix /. w → 0 // FullSimplify;
DMatrix = DMatrix /. w → 0 // FullSimplify;
K = Inverse[FullMatrix].DMatrix;
```

```
x4dotimpact = FullSimplify[K[[4]]]
x5dotimpact = FullSimplify[K[[5]]]
x6dotimpact = FullSimplify[K[[6]] + K[[7]]]
x7dotimpact = -FullSimplify[K[[7]]]
```

$$\frac{1}{M+4\,Mp}\left(\mathrm{Sec}\left[\frac{x7[t]}{2}\right]\left(\left(M\,\mathrm{Cos}\left[2\,x6[t]+\frac{3\,x7[t]}{2}\right]+2\,(M+2\,Mp)\,\mathrm{Sin}[x6[t]]\,\mathrm{Sin}\left[x6[t]+\frac{x7[t]}{2}\right]\right)x4'[t]+2\,((M+2\,Mp)\,\mathrm{Cos}[x6[t]]-M\,\mathrm{Cos}[x6[t]+x7[t]])\,\mathrm{Sec}[x5[t]]\,\mathrm{Sin}\left[x6[t]+\frac{x7[t]}{2}\right]x5'[t]\right)\right)$$

$$\frac{1}{M+4\,Mp}\left(\mathrm{Sec}\left[\frac{x7[t]}{2}\right]\left(2\,\mathrm{Cos}[x5[t]]\,\mathrm{Cos}\left[x6[t]+\frac{x7[t]}{2}\right]((M+2\,Mp)\,\mathrm{Sin}[x6[t]]-M\,\mathrm{Sin}[x6[t]+x7[t]])\,x4'[t]+\left(2\,(M+2\,Mp)\,\mathrm{Cos}[x6[t]]\,\mathrm{Cos}\left[x6[t]+\frac{x7[t]}{2}\right]-M\,\mathrm{Cos}\left[2\,x6[t]+\frac{3\,x7[t]}{2}\right]\right)x5'[t]\right)\right)$$

$$\frac{1}{2\,(M+4\,Mp)\,(-3\,M-4\,Mp+2\,M\,\mathrm{Cos}[2\,x7[t]])}\left(\mathrm{Sec}[x5[t]]\,\mathrm{Sec}\left[\frac{x7[t]}{2}\right]\left(\mathrm{Sin}[2\,x5[t]]\left((M+2\,Mp)\left(2\,(M+2\,Mp)\,\mathrm{Cos}\left[\frac{3\,x7[t]}{2}\right]+M\,\mathrm{Cos}\left[\frac{5\,x7[t]}{2}\right]\right)+\mathrm{Cos}\left[\frac{x7[t]}{2}\right](-M\,(3\,M+8\,Mp)+2\,\mathrm{Cos}[2\,x6[t]]\,((M+Mp)\,(3\,M+4\,Mp)+M\,(-2\,(M+2\,Mp)\,\mathrm{Cos}[x7[t]]-2\,(M+Mp)\,\mathrm{Cos}[2\,x7[t]]+M\,\mathrm{Cos}[3\,x7[t]])))-2\,(Mp\,(3\,M+4\,Mp)+M\,(-2\,(M+2\,Mp)\,\mathrm{Cos}[x7[t]]-2\,Mp\,\mathrm{Cos}[2\,x7[t]]+M\,\mathrm{Cos}[3\,x7[t]]))\,\mathrm{Sin}[2\,x6[t]]\,\mathrm{Sin}\left[\frac{x7[t]}{2}\right]\right)x4'[t]+2\left(2\,(-3\,M-4\,Mp+2\,M\,\mathrm{Cos}[2\,x7[t]])\,((M+2\,Mp)\,\mathrm{Cos}[x6[t]]-M\,\mathrm{Cos}[x6[t]+x7[t]])\,\mathrm{Sin}[x5[t]]\,\mathrm{Sin}\left[x6[t]+\frac{x7[t]}{2}\right]x5'[t]+(M+4\,Mp)\,\mathrm{Cos}[x5[t]]\,\mathrm{Cos}\left[\frac{x7[t]}{2}\right]((M-2\,(M+2\,Mp)\,\mathrm{Cos}[x7[t]])\,x6'[t]+M\,x7'[t])\right)\right)\right)$$

$$-\left(8\,(M+Mp)\,(1+2\,\mathrm{Cos}[x7[t]])\,\mathrm{Sin}\left[\frac{x7[t]}{2}\right]^2(\mathrm{Sin}[x5[t]]\,x4'[t]-x6'[t])+M\,(1-2\,\mathrm{Cos}[x7[t]])\,x7'[t]\right)\Big/(-3\,M-4\,Mp+2\,M\,\mathrm{Cos}[2\,x7[t]])$$

Collision Guard

Note: gamma is the slope angle

```
height = First[pNSFootp[[3]] + Tan[gamma] * pNSFootp[[2]] /. {x2[t] → 0, x3[t] → 0}]
```

```
-w Sin[x5[t]] + 2 l Cos[x5[t]] Sin[x6[t] + x7[t]/2] Sin[x7[t]/2] +
  (w Cos[x5[t]] Sin[x4[t]] + 2 l (Cos[x4[t]] Cos[x6[t] + x7[t]/2] +
        Sin[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]/2]) Sin[x7[t]/2]) Tan[gamma]
```

```
Avect = {{D[height, x4[t]], D[height, x5[t]], D[height, x6[t]], D[height, x7[t]]}};
holonomicTraj =
 First[First[Avect.{{x4'[t]}, {x5'[t]}, {x6'[t]}, {x7'[t]}}]] // FullSimplify
```

```
(w Cos[x4[t]] Cos[x5[t]] +
    2 l (-Cos[x6[t] + x7[t]/2] Sin[x4[t]] + Cos[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]/2])
     Sin[x7[t]/2]) Tan[gamma] x4′[t] -
  (Sin[x5[t]] (l Cos[x6[t]] - l Cos[x6[t] + x7[t]] + w Sin[x4[t]] Tan[gamma]) +
    Cos[x5[t]] (w - 2 l Sin[x4[t]] Sin[x6[t] + x7[t]/2] Sin[x7[t]/2] Tan[gamma])) x5′[t] +
 2 l Sin[x7[t]/2] (Cos[x5[t]] Cos[x6[t] + x7[t]/2] +
     (Cos[x6[t] + x7[t]/2] Sin[x4[t]] Sin[x5[t]] - Cos[x4[t]] Sin[x6[t] + x7[t]/2]) Tan[gamma])
  x6′[t] + l (Cos[x4[t]] Cos[x6[t] + x7[t]] Tan[gamma] +
    Sin[x6[t] + x7[t]] (Cos[x5[t]] + Sin[x4[t]] Sin[x5[t]] Tan[gamma])) x7′[t]
```

The guard is the zero-level set of the height function:

```
height == 0
```

```
-w Sin[x5[t]] + 2 l Cos[x5[t]] Sin[x6[t] + x7[t]/2] Sin[x7[t]/2] +
  (w Cos[x5[t]] Sin[x4[t]] + 2 l (Cos[x4[t]] Cos[x6[t] + x7[t]/2] +
         Sin[x4[t]] Sin[x5[t]] Sin[x6[t] + x7[t]/2]) Sin[x7[t]/2]) Tan[gamma] == 0
```

and the negative region of the holonomic constraint's trajectory:

holonomicTraj < 0

$$
\begin{aligned}
&\Big(\text{w Cos}[\text{x4}[\text{t}]]\,\text{Cos}[\text{x5}[\text{t}]] + \\
&\quad 2\,\text{l}\,\Big(-\text{Cos}\Big[\text{x6}[\text{t}] + \frac{\text{x7}[\text{t}]}{2}\Big]\,\text{Sin}[\text{x4}[\text{t}]] + \text{Cos}[\text{x4}[\text{t}]]\,\text{Sin}[\text{x5}[\text{t}]]\,\text{Sin}\Big[\text{x6}[\text{t}] + \frac{\text{x7}[\text{t}]}{2}\Big]\Big) \\
&\quad\ \text{Sin}\Big[\frac{\text{x7}[\text{t}]}{2}\Big]\Big)\,\text{Tan}[\text{gamma}]\,\text{x4}'[\text{t}] - \\
&\ \Big(\text{Sin}[\text{x5}[\text{t}]]\,(\text{l Cos}[\text{x6}[\text{t}]] - \text{l Cos}[\text{x6}[\text{t}] + \text{x7}[\text{t}]] + \text{w Sin}[\text{x4}[\text{t}]]\,\text{Tan}[\text{gamma}]) + \\
&\quad \text{Cos}[\text{x5}[\text{t}]]\,\Big(\text{w} - 2\,\text{l Sin}[\text{x4}[\text{t}]]\,\text{Sin}\Big[\text{x6}[\text{t}] + \frac{\text{x7}[\text{t}]}{2}\Big]\,\text{Sin}\Big[\frac{\text{x7}[\text{t}]}{2}\Big]\,\text{Tan}[\text{gamma}]\Big)\Big)\,\text{x5}'[\text{t}] + \\
&\ 2\,\text{l Sin}\Big[\frac{\text{x7}[\text{t}]}{2}\Big]\,\Big(\text{Cos}[\text{x5}[\text{t}]]\,\text{Cos}\Big[\text{x6}[\text{t}] + \frac{\text{x7}[\text{t}]}{2}\Big] + \\
&\quad \Big(\text{Cos}\Big[\text{x6}[\text{t}] + \frac{\text{x7}[\text{t}]}{2}\Big]\,\text{Sin}[\text{x4}[\text{t}]]\,\text{Sin}[\text{x5}[\text{t}]] - \text{Cos}[\text{x4}[\text{t}]]\,\text{Sin}\Big[\text{x6}[\text{t}] + \frac{\text{x7}[\text{t}]}{2}\Big]\Big) \\
&\quad\ \text{Tan}[\text{gamma}]\Big)\,\text{x6}'[\text{t}] + \text{l}\,(\text{Cos}[\text{x4}[\text{t}]]\,\text{Cos}[\text{x6}[\text{t}] + \text{x7}[\text{t}]]\,\text{Tan}[\text{gamma}] + \\
&\quad \text{Sin}[\text{x6}[\text{t}] + \text{x7}[\text{t}]]\,(\text{Cos}[\text{x5}[\text{t}]] + \text{Sin}[\text{x4}[\text{t}]]\,\text{Sin}[\text{x5}[\text{t}]]\,\text{Tan}[\text{gamma}]))\,\text{x7}'[\text{t}] < 0
\end{aligned}
$$