Highway Traffic Flow Analysis and Control

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Abstract

Significant inspiration of computer network topology and communication comes from an empirical understanding of how road networks function. This project takes foundational work by CHESS researchers in hybrid systems to investigate a macroscopic switching-mode model (SMM) of traffic. The goal is to learn how hybrid systems are used for traffic modeling, experiment with different techniques for reachability analysis, implement a controller for the system, and then study the system behavior in the presence of disturbances. The end result will be MATLAB simulations which show the impact of the work.

1 Introduction

Highways are becoming increasingly congested as the use of cars continues to gain popularity. This phenomenon requires finding methods of optimizing traffic flow, especially during rush hours. One of the most popular methods for controlling highway traffic is the use of on-ramp metering. This method restricts the rate at which vehicles enter the highway by means of a traffic light. The first on-ramp metering occurred in Chicago in the early 1960's on the Chicago Eisenhower Expressway [9].

Figure 1. On-ramp Metering

Most ramp meters create a four to fifteen second delay between the cars entering the highway though some meters allow more than one car to pass at a time. If cars enter the highway in controlled intervals, they are less likely to cause a major traffic disruption. Furthermore, on-ramp metering has been shown to increase the speed of cars on the highway by up to fifty percent and reduce highway accidents by more than thirty percent in some cases, according to the Washington State Department of Transportation. Moreover, empirically it has been shown that no more than five to ten percent of vehicles will be diverted because of on-ramp metering [9].

The Freeway Performance Measurement System (PeMS) is a project in Berkeley's EECS Department in collaboration with the California Department of Transportation (Caltrans), the California Partners for Advanced Transit and Highways (PATH), and the Berkeley Transportation Systems. This system collects and stores historical and real-time data from the California freeways as well as provides performance measures. The data is collected from the information received by the sensor loops that count the number of vehicles on the road at any time and the speed of the vehicles. California is divided into twelve districts, so this aids in organizing the collected data. As of July 2005, real-time data was being collected from six Caltrans districts and from 26,000 loops, providing approximately 2GB of data per day, resulting in a total of 5.4TB of data on the website [3]. Anyone can register for an account with PeMS to access the data and obtain the tools required for analysis. On the website, one can access different highways and assess data at specific times. This method is used to identify bottlenecks. We can look at contour plots for either density or speed of vehicles and find where the bottlenecks are located [6].

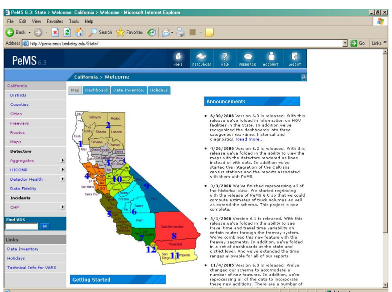

Figure 2. 12 Districts of California as seen on PeMS website

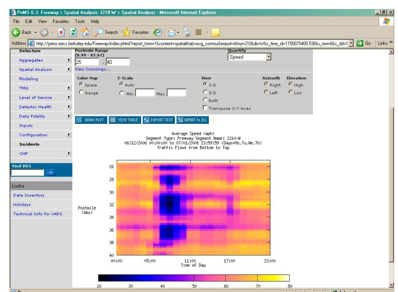

Figure 3. Spatial Analysis depicting bottlenecks on PeMS website

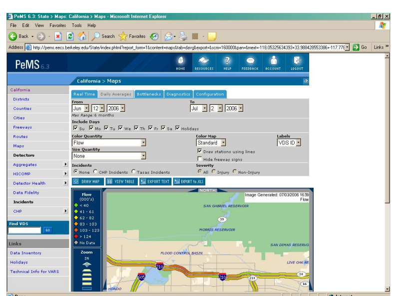

Figure 4. I-210 from PeMS website

There are two different types of models that are considered when dealing with traffic: the microscopic traffic model and the macroscopic traffic model. In the microscopic traffic model, dynamics of individual cars and the interactions between them and their surroundings are explicitly modeled. Behaviors such as a vehicle following other vehicles and changing lanes are studied. These models typically provide better analysis of traffic behavior than macroscopic models, but the computations are much more complex. Conversely, in the macroscopic traffic models, traffic is treated as a continuum and modeled by aggregated, fluid-like quantities, such as density, flow, and speed [4]. Therefore, the computations are more general and simpler. Partial differential equations are often used to describe the changes in these quantities over time. In this paper, we will be considering macroscopic traffic models.

The focus of this research will be on Interstate 210 westbound in Pasadena, California in the direction of downtown Los Angeles. There is heavy traffic on this stretch of highway, especially during rush hours, which occur on weekday mornings and daily evenings. Highways become congested as a result of various factors. The most common causes of bottlenecks are off-ramp backup, traffic merging from the on-ramps, accidents, and a fork in the highway. We will be concentrating on the bottlenecks that are due to cars merging onto the highway from on-ramps. Then we will use on-ramp metering to control the traffic and try to optimize automobile density and flow using different models.

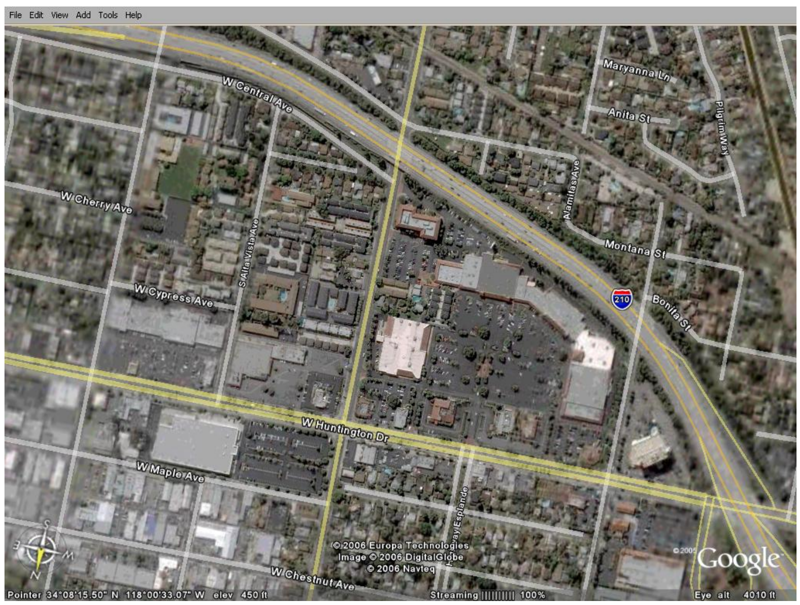

Figure 5. I-210W (section considered for simulation)

2 Two Models: CTM and SMM

There are two models that will be considered for this project, and they are the Cell Transmission Model and the Switching-Mode Model.

2.1 Cell Transmission Model (CTM)

The Cell Transmission Model was first developed by Carlos F. Daganzo, Berkeley, in 1993 [1]. Each station that is associated with a cross-street from Caltrans is placed in the middle of a cell. The cells, described individually, are used in the model.

$$\rho_i(k+1) = \rho_i(k) + \frac{T_s}{l_i}(q_i(k) - q_{i+1}(k)) \tag{1}$$

The above equation 1 is the recursion equation that is used in the CTM, where k is the time index, T_s is the discrete time interval, l_i is the length of cell i, and $q_i(k)$ is the flow rate, in vehicles per unit time, into cell i during the interval [k, k+1) [7]. To find $q_i(k)$, we take the minimum of two quantities:

$$q_i(k) = min(S_{i-1}(k), R_i(k)) \tag{2}$$

where $S_{i-1}(k) = min(v\rho_{i-1}(k), Q_{M,i-1})$ is the maximum flow that can be supplied by cell i-1 in the free-flow mode, over the interval [k,k+1), and $R_i(k) = min(Q_{M,i}w(\rho_J - \rho_i(k)))$ is the maximum flow that can be received by cell i in the congested mode, over the same time interval [7]. The quantities are found using the Fundamental Diagram (Figure 10). This Diagram depicts the dynamic information specific to a particular macroscopic model and the relation between vehicle flux and vehicle density. It shows that vehicles on a highway are in free-flow up to a certain point, at which there is a plateau (maximum capacity on the highway is reached). After the plateau, there is highway congestion until vehicles completely stop moving. The process with this model is to take real data, split the highway into cells, then apply the cells and the diagram. One of the main disadvantages of this method is that it is a nonlinear model so the analyis methods are more complex. A continuation of Daganzo's CTM research may be found in [2].

2.2 Switching-Mode Model (SMM)

The Switching-Mode Model is a piecewise-linearized version of the cell transmission model, and it was developed by Laura Muñoz et al [4]. It simplifies system analysis, data estimation, and control design. There is one linear system for free flow and one for congestion. The segment of highway consists of cells; the whole segment is either in free flow or congested state. A hybrid system is a dynamical system with both discrete and continuous state changes, so the switch between the two states (free flow and congestion) creates a hybrid system. In this model, the aforementioned Fundamental Diagram is replaced with a set of mode-transition rules that depend on the congestion of cells and the mainline boundary data. The SMM is good for estimation, but it is not quite clear how good it is for control, and additionally, the boundary is a problem if and when a switch between two states occurs. The biggest advantage of the SMM is the fact that it possesses a linear mode structure allowing for linear analysis methods, which are less complex and less time consuming. The accuracy and computation time is comparable to that of the CTM [6].

2.2.1 Mode Equation Structure

This is how the section of highway considered is divided into cells and the set of equations that are used in the Switching-Mode Model [5].

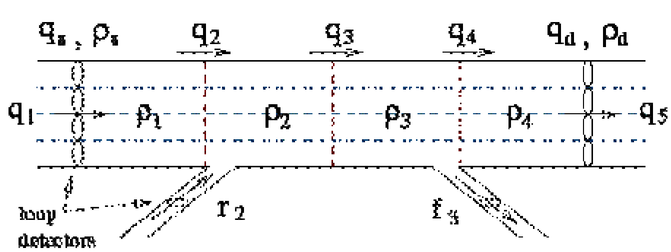

Figure 6. Cells on section of highway

$$\rho = [\rho_1 ... \rho_4]^T \tag{3}$$

$$u = [q_u r_2 \rho_d]^T \tag{4}$$

$$\rho_J = [\rho_J 1 \rho_J 2 \rho_J 3 \rho_J 4 \rho_J 5]^T \tag{5}$$

$$q_M = [Q_{M1} Q_{M2} Q_{M3} Q_{M4}]^T \tag{6}$$

$$\rho(k+1) = A_s(k)\rho(k) + B_s u(k) + B_{J,s}(k)\rho_J + B_{Q,s} q_M \tag{7}$$

In these equations, s=1,2,3,4,5 indicates the mode, ρ represents traffic density whereas Q represents flow [7]. The recursive equation (7) is obtained by the values obtained from the four equations listed before it. Then A and B are taken from the following matrices after it is decided if there is free-flow or congestion.

2.2.2 Free-flow Model

Above we see the matrices that are used in (7) for the case when vehicles are in free-flow.

$$A_1(k) = \begin{bmatrix} 1 - \frac{v_1 T_s}{l_1} & 0 & 0 & 0 \\ \frac{v_1 T_s}{l_2} & 1 - \frac{v_2 T_s}{l_2} & 0 & 0 \\ 0 & \frac{v_2 T_s}{l_3} & 1 - \frac{v_3 T_s}{l_3} & 0 \\ 0 & 0 & (1 - \beta(k))\frac{v_3 T_s}{l_4} & 1 - \frac{v_4 T_s}{l_4} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \frac{T_s}{l_1} & 0 & 0 \\ 0 & \frac{T_s}{l_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_{J,1} = 0_{4\times5}; \quad B_{Q,1} = 0_{4\times4}.$$

Figure 7. Free-Flow Model

2.2.3 Congestion Model

Figure 8 shows the matrices that are used in (7) when the highway is in the congested mode.

3 Simulation

The goal is to determine if on-ramp metering reduces travel time in traffic congestion delay. A simulation of a short stretch of I-210 westbound is created in MATLAB and divided into four cells, including an on-ramp and an off-ramp. The simulation measures traffic density, flow, and overall travel time.

$$A_2(k) = \begin{bmatrix} 1-\frac{w_1T_s}{l_1} & \frac{w_2T_s}{l_1} & 0 & 0 \\ 0 & 1-\frac{w_2T_s}{l_2} & \frac{w_3T_s}{l_2} & 0 \\ 0 & 0 & 1-\frac{w_3T_s}{l_3} & \frac{1}{1-\beta(k)}\frac{w_4T_s}{l_3} \\ 0 & 0 & 0 & 1-\frac{w_4T_s}{l_4} \end{bmatrix}, B_2 = \begin{bmatrix} 0 & \frac{T_s}{l_1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{w_5T_s}{l_4} \end{bmatrix},$$

$$B_{J,2}(k) = \begin{bmatrix} \frac{w_1T_s}{l_1} & -\frac{w_2T_s}{l_1} & 0 & 0 & 0 \\ 0 & \frac{w_2T_s}{l_2} & -\frac{w_3T_s}{l_2} & 0 & 0 \\ 0 & 0 & \frac{w_3T_s}{l_3} & -\frac{1}{1-\beta(k)}\frac{w_4T_s}{l_3} & 0 \\ 0 & 0 & 0 & \frac{w_4T_s}{l_4} & -\frac{w_5T_s}{l_4} \end{bmatrix},$$

$$B_{Q,2} = 0_{4\times 4}.$$

Figure 8. Congestion Model

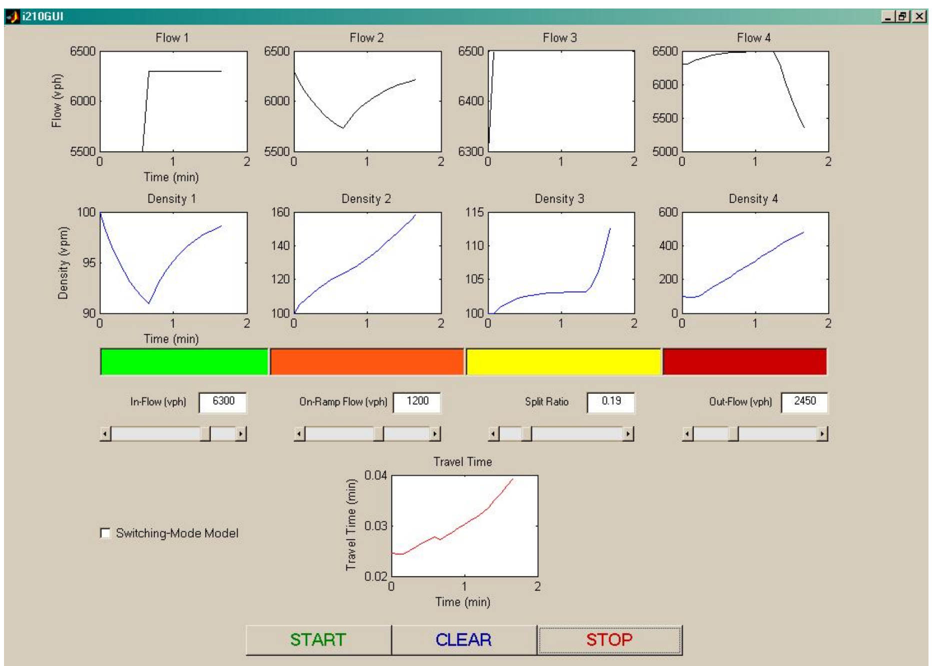

Figure 9. Screen Capture of MATLAB GUI Simulation

3.1 Highway–›Simulation

One stretch of highway from I-210 westbound was chosen to be used in the simulation. The section is over one mile long and by the Huntington intersection (the cross street given by Caltrans). This part of the highway was picked because of the heavy traffic that is experienced on it especially during rush hours and for the reason that it has an on-ramp and an off-ramp. The simulation was made using a GUI in MATLAB, and the section of highway is divided into four cells. The first cell represents the inflow of traffic, the second is the cell with the on-ramp, the third cell has the off-ramp, and the last cell is for the outflow of traffic.

Each cell has the initial conditions: 5500 vehicles per hour for in-flow, 1200 vehicles per hour entering from

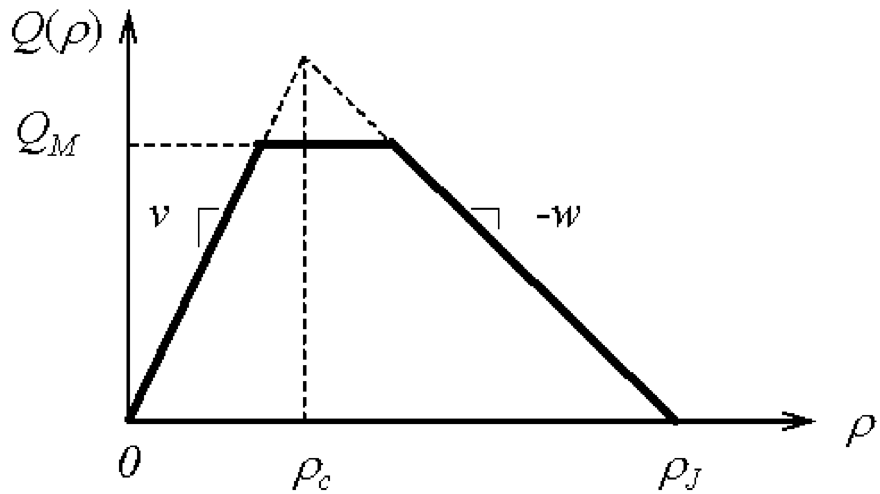

Figure 10. Fundamental Diagram. Here, v: free-flow speed (mph), w: backward congestion wave speed (mph), Q_M: maximum allowable flow (vph), ρ_J: jam density (vpm), ρ_c: critical density (vpm)

the on-ramp, .19 as the split ratio (ratio of cars exiting the highway by means of the off-ramp), and 5650 vehicles per hour for the out-flow. Then if we run the simulation with this set of initial conditions without altering them, all of the cells will be green except for the second one with the on-ramp. This one will initially be yellow then progressively worsen to an orange and then red color. It will finally affect the traffic on both the first and second cell, placing them in congestion, and increasing overall travel time. The colors represent the different locations on Daganzo's Fundamental Diagram. Green represents the first section of the diagram of free-flow before the plateau is reached. Yellow represents the first part of the plateau up to the critical density. Orange represents the second part of the plateau after the critical density mark but before congestion and the downward slope. Red represents the last section of the diagram that slopes down, indicating congestion until jam density (the point where no cars are moving).

The initial conditions can be altered by moving the arrows below the numbers or by replacing the numbers by new ones. Above the cells are two sets of graphs. The first one plots density for each cell, measuring vehicles per mile by time in minutes. The graph on the top plots the flow for each cell, measuring vehicles per hour by time in minutes. We are most interested in minimizing the overall travel time from the beginning of the first cell to the end of the fourth cell by looking at the graph plotted at the bottom below the cells. This shows the overall travel time on the highway at the real time (of the simulation).

3.2 Uses of the Two Models to Control On-ramps

The simulation is set to use the Cell-Transmission Model and the fundamental diagram. We can also switch to the Switching-Mode Model and then we have only two colors representing free flow (green) and congestion (red) in each cell. This is because in SMM, we no longer use the Fundamental Diagram; instead it is made up of two linear systems.

4 Conclusion and Further Work

On-ramp metering is a successful and efficient tool in reducing traffic on highways. The simulation helped to visualize how on-ramp metering works in practice and to try various methods of on-ramp metering. The CTM model was used to help predict traffic flow (and therefore, also density), which can aid in developing more efficient strategies in on-ramp metering. In the future, we would like to decide which model is better for controlling on-ramps: CTM or SMM by studying these two models more closely and using methods to compare them and optimize the on-ramp metering using one of these models.

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