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SUPERB-IT

Center for Hybrid and Embedded Software Systems

# Highway Traffic Flow Analysis and Control

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## Abstract

This project takes foundational work by CHES researchers in hybrid systems to investigate a macroscopic switching-mode model (SMM) of traffic. The goal is to learn how hybrid systems are used for traffic modeling, experiment with different techniques for reachability analysis, implement a controller for the system, and then study the system behavior in the presence of disturbances.

## Motivation

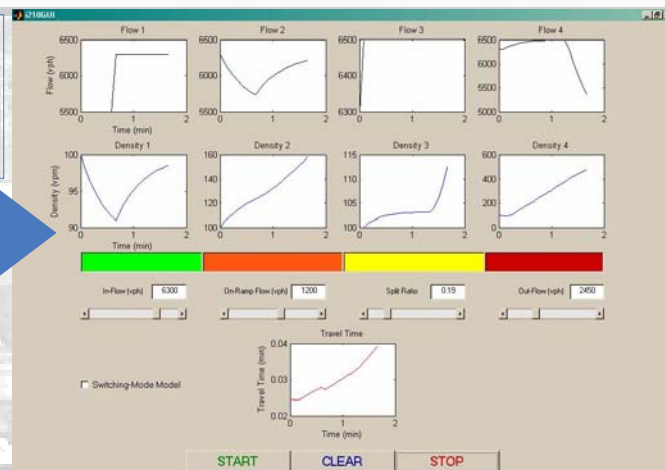
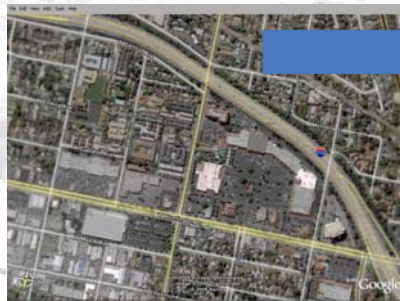
On-ramp metering is a popular method for controlling highway traffic (which is becoming a greater problem) and has been shown to allay traffic. We need to optimize how on-ramps control traffic to minimize travel time.

## Methods

- Traffic measured by sensor loops that count number of vehicles, speed, and number of cars on freeway (data collected by PEMS), 12 districts in CA.
- Macroscopic Traffic Model: traffic is treated as a continuum and modeled by aggregated, fluid-like quantities, i.e., density, flow, and speed (PDEs are often used to describe the changes in these quantities over time)
- Two Models to control highway traffic: CTM and SMM

## Results

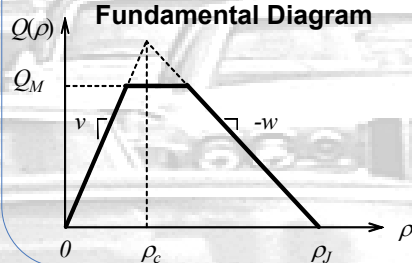
MATLAB Simulation covers a short stretch of traffic (I-210 westbound), divided into 4 sections, with on-ramp and off-ramp, measuring traffic density, flow, and travel time.



## CTM (Cell-Transmission Model) Equation

$$\rho_i(k+1) = \rho_i(k) + \frac{T_s}{L_i}(q_i(k) - q_{i+1}(k))$$

## Fundamental Diagram



- $v$ : free-flow speed (mph)
- $w$ : backward congestion wave speed (mph)
- $Q_M$ : maximum allowable flow (vph)
- $\rho_j$ : jam density (vpm)
- $\rho_c$ : critical density (vpm)

## SMM (Switching-Mode Model):

Piecewise-linearized version of CTM  
One linear system for free flow, one for congestion

## Mode Equation Structure

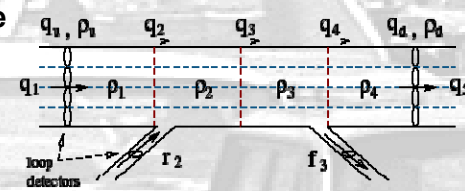
$$\rho = [\rho_1 \dots \rho_4]^T$$

$$u = [q_u \tau_2 \rho_d]^T$$

$$\rho_J = [\rho_{J1} \rho_{J2} \rho_{J3} \rho_{J4} \rho_{J5}]^T$$

$$q_M = [q_{M1} \ q_{M2} \ q_{M3} \ q_{M4}]^T$$

$$\rho(k+1) = A_s(k) \rho(k) + B_s u(k) + B_{J,s}(k) \rho_J + B_{Q,s} q_M$$



## Free-Flow Model

$$A_1(k) = \begin{bmatrix} 1 - \frac{v_1 T_s}{L_1} & 0 & 0 & 0 \\ \frac{v_1 T_s}{L_1} & 1 - \frac{v_2 T_s}{L_2} & 0 & 0 \\ 0 & \frac{v_2 T_s}{L_2} & 1 - \frac{v_3 T_s}{L_3} & 0 \\ 0 & 0 & \frac{v_3 T_s}{L_3} & 1 - \frac{v_4 T_s}{L_4} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{J,1} = 0_{4 \times 5}, \quad B_{Q,1} = 0_{4 \times 4}$$

## Congestion Model

$$A_2(k) = \begin{bmatrix} 1 - \frac{w_1 T_s}{L_1} & \frac{w_1 T_s}{L_1} & 0 & 0 \\ 0 & 1 - \frac{w_2 T_s}{L_2} & 0 & 0 \\ 0 & 0 & 1 - \frac{w_3 T_s}{L_3} & 0 \\ 0 & 0 & 0 & 1 - \frac{w_4 T_s}{L_4} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{J,2} = \begin{bmatrix} \frac{v_1 T_s}{L_1} & \frac{v_2 T_s}{L_2} & 0 & 0 \\ 0 & \frac{v_2 T_s}{L_2} & 0 & 0 \\ 0 & 0 & \frac{v_3 T_s}{L_3} & 0 \\ 0 & 0 & 0 & \frac{v_4 T_s}{L_4} \end{bmatrix}$$

$$B_{Q,2} = 0_{4 \times 4}$$

## Future Research

Use SMM (Switching-Mode Model) for Control using hybrid systems and determine if on-ramp metering reduces travel time in traffic congestion delay

## References

- Daganzo, C.V. The Cell Transmission Model: A Dynamic Representation of Highway Traffic Consistent with the Hydrodynamic Theory. *Transportation Research – B*, Vol. 28, No. 4, 1994, pp. 269-287.
- Muñoz, Laura, Sun, Xiaotian, Horowitz, Roberto, Alvarez, Luis. *Review of Switching Mode Model and Parameter Calibration Method*. April 28, 2006.

