EE249 Discussion: Petri Net

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Outline

● Basics of Petri Net
● Applications of Petri Net
  ○ Supply Chain Modeling
  ○ Biological Network
● Petri Net Synthesis
Basics of Petri Net
Petri Net Recap

- A directed-bipartite graph
- Two kinds of nodes:
  a. places (-> resources)
  b. transitions (-> events)
- Test arc

- Inhibitor arc
Analysis of Petri Net

**pre-condition matrix**

<table>
<thead>
<tr>
<th></th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_2$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_3$</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

**post-condition matrix**

<table>
<thead>
<tr>
<th></th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$p_3$</td>
<td>4</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

**incidence matrix**

<table>
<thead>
<tr>
<th></th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_2$</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$p_3$</td>
<td>4</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

**initial marking**

$M_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

**state equation**

$M' = M + C \cdot \sigma,$

$M_2 = M_0 + C \cdot \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$
Petri Net Recap

- Boundedness: # of tokens is always bounded
- S/P-invariant: set of places which total token count remains constant
- T-invariant: sequence of transitions that bring back to original marking
- Reachability: able to evolve from initial marking to targeted marking
- Liveness: always able to fire any transition
Petri Net Applications
Petri Net on Supply Chains

- Used generalised stochastic Petri nets (GSPN) models for analysis
- Solve two cost minimisation problems:
  a. comparison of make-to-stock (MTS) and assemble-to-order (ATO) policies
  b. locating decoupling point in supply chain
- Software used: stochastic petri net package (SPNP)
Supply Chain

suppliers

inbound logistics

OEM

outbound logistics

warehouses
MTS vs ATO: Setup

- $H_i$: incurred holding cost for inventories of first stage (i.e. A and B)
- $H_D$: cost per hour of delayed delivery
MTS vs ATO: Setup

- Assumptions:
  - \( H_D \) of \( E \) = 150% \( H_D \) of \( D \)
  - \( H_I \) of \( E \) or \( D \) = 120% \( H_I \) of \( C \);
    \[ H_I \text{ of } C = 120\% \quad H_I \text{ of } A \text{ or } B \]
- \( I_A, I_B, I_C = 6 \) units; \( I_D, I_E = 3 \) units (for MTS only)
- Inventory level that triggers reorder:
  - \( A, B = 1 \) unit; \( C = 10 \) units; \( D, E = 1 \) unit (for MTS only)
MTS vs ATO: Setup

- Transition firing rate:

<table>
<thead>
<tr>
<th>Transition name</th>
<th>Firing rate (units/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1</td>
<td>1.00</td>
</tr>
<tr>
<td>t2</td>
<td>1.00</td>
</tr>
<tr>
<td>t3</td>
<td>3.00</td>
</tr>
<tr>
<td>t4</td>
<td>2.00</td>
</tr>
<tr>
<td>t5</td>
<td>6.00</td>
</tr>
<tr>
<td>t6</td>
<td>4.00</td>
</tr>
<tr>
<td>t8</td>
<td>2.00</td>
</tr>
<tr>
<td>t11</td>
<td>4.00</td>
</tr>
<tr>
<td>t12</td>
<td>2.00</td>
</tr>
<tr>
<td>t13</td>
<td>4.00</td>
</tr>
<tr>
<td>t14</td>
<td>3.00</td>
</tr>
<tr>
<td>t17</td>
<td>0.80</td>
</tr>
<tr>
<td>t18</td>
<td>0.60</td>
</tr>
</tbody>
</table>
MTS vs ATO: Effect of Arrival Rates

- Variation of total cost with arrival rates of D

<table>
<thead>
<tr>
<th>λ_D units/h</th>
<th>MTS system</th>
<th>ATO system</th>
<th>MTS system</th>
<th>ATO system</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>22.421</td>
<td>19.815</td>
<td>26.001</td>
<td>257.437</td>
</tr>
<tr>
<td>1.0</td>
<td>21.237</td>
<td>18.610</td>
<td>25.818</td>
<td>237.559</td>
</tr>
<tr>
<td>1.2</td>
<td>20.012</td>
<td>17.714</td>
<td>25.961</td>
<td>224.228</td>
</tr>
<tr>
<td>1.4</td>
<td>18.774</td>
<td>17.016</td>
<td>26.339</td>
<td>214.675</td>
</tr>
</tbody>
</table>
MTS vs ATO: Effect of Inventory

- FGI: finished goods inventory

### MTS

<table>
<thead>
<tr>
<th>$FGI_C$</th>
<th>$H_D/H_I = 1.5$</th>
<th>$H_D/H_I = 40.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>18.54</td>
<td>28.01</td>
</tr>
<tr>
<td>9</td>
<td>27.53</td>
<td>29.34</td>
</tr>
<tr>
<td>12</td>
<td>35.553</td>
<td>42.175</td>
</tr>
<tr>
<td>15</td>
<td>43.403</td>
<td>49.929</td>
</tr>
</tbody>
</table>

134%  

### ATO

<table>
<thead>
<tr>
<th>$FGI_C$</th>
<th>$H_D/H_I = 1.5$</th>
<th>$H_D/H_I = 40.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15.64</td>
<td>197.40</td>
</tr>
<tr>
<td>6</td>
<td>18.37</td>
<td>201.52</td>
</tr>
<tr>
<td>7</td>
<td>21.07</td>
<td>204.87</td>
</tr>
<tr>
<td>8</td>
<td>23.73</td>
<td>207.92</td>
</tr>
</tbody>
</table>

78%
MTS vs ATO: Effect of Interface Times of B

- Total costs increase marginally for both policies as interface times decreases
- Inventory held at C increases, as adds more inventory at greater pace -> more holding costs
- Conclusion: need to consider the whole supply chain (for e.g., need to reduce interface times at A too to derive benefits)

<table>
<thead>
<tr>
<th>Interface rates with $S_2$ units/h</th>
<th>$H_D/H_I = 1.5$</th>
<th>$H_D/H_I = 40.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MTS system</td>
<td>ATO system</td>
</tr>
<tr>
<td>4.0</td>
<td>22.566</td>
<td>15.542</td>
</tr>
<tr>
<td>5.0</td>
<td>22.651</td>
<td>15.640</td>
</tr>
<tr>
<td>6.0</td>
<td>22.709</td>
<td>15.705</td>
</tr>
<tr>
<td>8.0</td>
<td>22.780</td>
<td>15.785</td>
</tr>
</tbody>
</table>
Decoupling Point Location Problem (DPLP)

- Decoupling point (DP): point in space where order is assigned to the customer
- Aim: to find minimal cost DP
  a. cost of holding inventory before DP
  b. cost of excess lead time from DP onwards
Decoupling Point Location Problem (DPLP)

Figure 7: The aggregated GSPN model of the supply chain considered for DPLP.
DPLP Setup

- Use an integrated queuing-GSPN model
- Holding costs increase by 20% each stage as move away to distribution centre

<table>
<thead>
<tr>
<th>Facility</th>
<th>Average service rate (jobs/h) ( \mu )</th>
<th>SCV of service time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>10.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>15.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Initial assembly plant</td>
<td>25.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Final assembly plant</td>
<td>10.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Distribution</td>
<td>20.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>
DPLP Analysis

- DP moves to the right (i.e. toward the retail outlets) as $H_D/H_I$ increases.
Petri nets for Biological Networks
Overview

- Summary paper to raise awareness
- "Petri Net Basics" section summarizes class
- Table 1, available software tools
- 50+ references to specific BioPN papers
- Outlines a few ways PN structures can describe biological processes
Figure 2, PN structures for biological reactions
Note: complex formation, binding process, transcriptional regulation, etc.
Applicability

- PNs a significant tool for practical biological abstraction
- Wide selection in software analysis tools
- PNs well suited to the qualitative types of data available in biological sciences
  - Relation, not concentration
- Boundedness: in biology, means no product can accumulate
- T-invariants: could represent a cyclical metabolic process
New PN types

- **Stochastic Petri Nets (SPNs)**
  - Random molecular interaction rates
- **Hybrid Petri Nets (HPNs)**
  - Allowing continuous and discrete time
- **Functional Petri Nets (FPNs)**
  - Including state-dependent control logic
- **Hybrid Functional Petri Nets (HFPNs)**
  - Introduced specifically for biological modeling
- **Colored Petri Nets (CPNs)**
  - Multiple types of tokens

Most biological models can be unified into some type of Petri Net
Sample applications

- Genetic networks
  - Standard PNs possible, but CPNs enable a compact model and automated analysis
  - HPNs and HFPNs also considered, but data to create kinetic models is lacking

- Signaling networks
  - HFPNs, standard PNs, timed PNs, timed CPNs all considered to model certain aspects

- Many viable options exist to model given processes
Petri Net Synthesis/Scheduling
Intro

- Paper focuses on the scheduling problem
  - Only necessary for concurrent use of shared, limited resources
- Three steps to scheduling/synthesis:
  - Static, with truly fixed dependencies
  - Quasi-static, with periodic dependencies
  - Dynamic, with dependencies decided at runtime
- If the dynamic tasks are schedulable, the algorithm decomposes it into many quasi-static states
PN review section

- Yet another summary of Petri Nets
- Much more technical overview
- Paper focus is on Free Choice Petri Nets
  - Each transition has only one predecessor
Schedulability?

- Only schedulable if rules aren't broken
  - Every transition is fired
  - No unbounded accumulation of tokens
  - Playing against adversary: anti-game theory

- Finding a schedule
  - Decompose into Conflict Free components (s. 77)
    - T-allocations such that only one transition is fired
    - T-reductions eliminate inactive net portion
  - Check if every component is statically schedulable
    - When looping back to a given system state/marking, ensure no deadlock
  - Schedule a loop that returns to the initial marking
Application

● Generate C-code from schedule
  ○ Insert code for blocks: ABBCC, ABCBC, etc.
  ○ Lines of C scales linearly with PN size
    ■ Generating schedule is exponential/polynomial
  ○ if-else: traverses both possibilities
  ○ multirate nets: counter determines how to proceed

● Over partitioning tasks by hand:
  ○ 23% savings in lines of code
  ○ 21% savings in clock cycles
  ○ 1000% savings in designer time
Conclusions

- Items (1) through (4) in last paragraph
  - Maximizes work done at compile time
  - Free-Choice PNs are decidable
  - Schedulability is verified, not assumed
  - Functions with same executions rates are combined
Thank you!