Synchronous Control and State Machines in Modelica

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MODELICA

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Introduction

- Why synchronous features in Modelica 3.3?

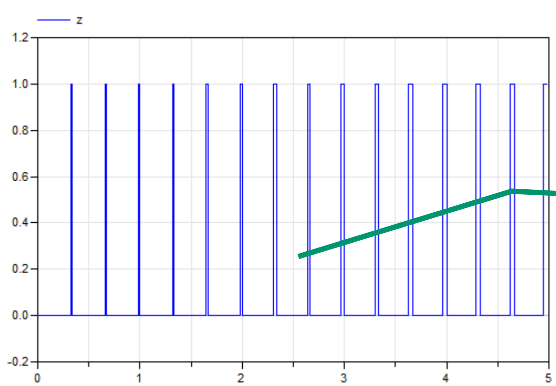
```
model Asynchronous_Modelica32
  Real x(start=0,fixed=true),
    y(start=0,fixed=true), z;
equation
  when sample(0,0.33) then
    x = pre(x)+1;
  end when;
  when sample(0,1/3) then
    y = pre(y)+1;
  end when;
  z = x-y;
end Asynchronous_Modelica32;
```

Implicit hold

```
model Asynchronous_Modelica33
  Real x(start=0,fixed=true),
    y(start=0,fixed=true), z;
equation
  when Clock(0.33) then
    x = previous(x)+1;
  end when;
  when Clock(1,3) then
    y = previous(y)+1;
  end when;
  z = x-y;
end Asynchronous_Modelica33;
```

Rational number 1/3

x and y must have the same clock



z = x-y

```
A subclock partition includes clocks that cannot be deduced to be equal.
Clock(1, 3)
Clock(0.33)
appears in the partition
when Clock_1 then
y = previous(y)+1;
end when;

when Clock_0 then
x = previous(x)+1;
end when;

z = x-y;
```

- Error Diagnostics for safer systems!

Introduction

- Scope of Modelica extended
- Covers complete system descriptions including controllers

- Clocked semantics
- Clock associated with variable type and inferred
- For increased correctness
- Based on ideas from Lucid Synchrone and other synchronous languages
- Extended with multi-rate periodic clocks, varying interval clocks and Boolean clocks

Synchronous Features of Modelica

- Plant and Controller Partitioning
- Boundaries between continuous-time and discrete-time equations defined by operators.
- sample(): samples a continuous-time variable and returns a clocked discrete-time expression
- hold(): converts from clocked discrete-time to continuous-time by holding the value between clock ticks
- sample operator may take a Clock argument to define when sampling should occur

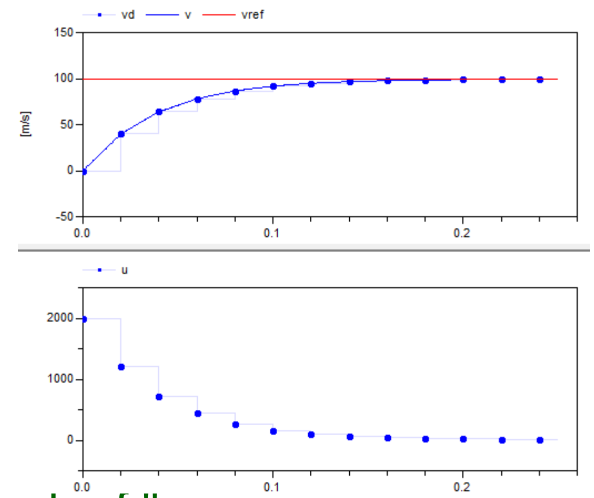
Mass with Spring Damper

- Consider a continuous-time model

```
partial model MassWithSpringDamper
  parameter Modelica.SIunits.Mass m=1;
  parameter Modelica.SIunits.TranslationalSpringConstant k=1;
  parameter Modelica.SIunits.TranslationalDampingConstant d=0.1;
  Modelica.SIunits.Position x(start=1,fixed=true) "Position";
  Modelica.SIunits.Velocity v(start=0,fixed=true) "Velocity";
  Modelica.SIunits.Force f "Force";
equation
  der(x) = v;
  m*der(v) = f - k*x - d*v;
end MassWithSpringDamper;
```

Synchronous Controller

- Discrete-time controller




```
model SpeedControl
  extends MassWithSpringDamper;
  parameter Real K = 20 "Gain of speed P controller";
  parameter Modelica.SIunits.Velocity vref = 100 "Speed ref.";
  discrete Real vd;
  discrete Real u(start=0);
equation
  // speed sensor
  vd = sample(v, Clock(0.01));

  // P controller for speed
  u = K*(vref-vd);

  // force actuator
  f = hold(u);
end SpeedControl;
```

Sample continuous velocity v
with periodic Clock with period=0.01

The clock of the equation
is inferred to be the same as for the variable
vd which is the result of sample()

Hold discrete variable u
between clock ticks

Discrete-time State Variables

- Operator previous() is used to access the value at the previous clock tick (cf pre() in Modelica 3.2)
- Introduces discrete state variable
- Initial value needed

- interval() is used to inquire the actual interval of a clock

Base-clocks and Sub-clocks

- A Modelica model will typically have several controllers for different parts of the plant.
- Such controllers might not need synchronization and can have different *base clocks*.
- Equations belonging to different base clocks can be implemented by asynchronous tasks of the used operating system.
- It is also possible to introduce sub-clocks that tick a certain factor slower than the base clock.
- Such sub-clocks are perfectly synchronized with the base clock, i.e. the definitions and uses of a variable are sorted in such a way that when sub-clocks are activated at the same clock tick, then the definition is evaluated before all the uses.
- New base type, Clock:

```
Clock cControl = Clock(0.01);
Clock cOuter = subSample(cControl, 5);
```

Sub and super sampling and phase

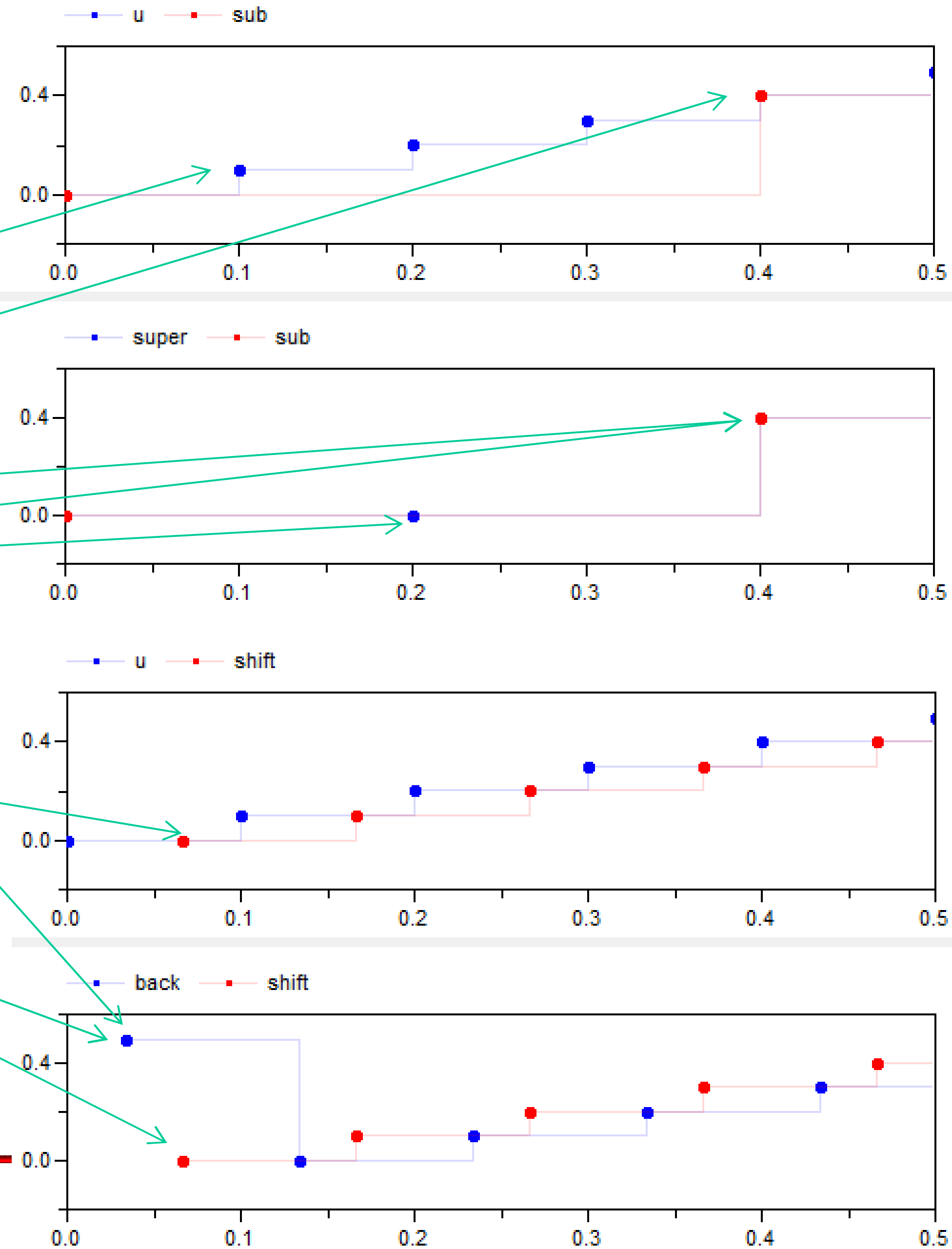
```
model SynchronousOperators
  Real u;

  Real sub;
  Real super;

  Real shift(start=0.5);
  Real back;
equation
  u = sample(time, Clock(0.1));

  sub = subSample(u, 4);
  super = superSample(sub, 2);

  shift = shiftSample(u, 2, 3);
  back = backSample(shift, 1, 3);
end SynchronousOperators;
```



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Exact Periodic Clocks

- Clocks defined by Real number period are not synchronized:

```
Clock c1 = Clock(0.1);
Clock c2 = superSample(c1,3);
Clock c3 = Clock(0.1/3);  // Not synchronized with c2
```

- Clocks defined by rational number period are synchronized:

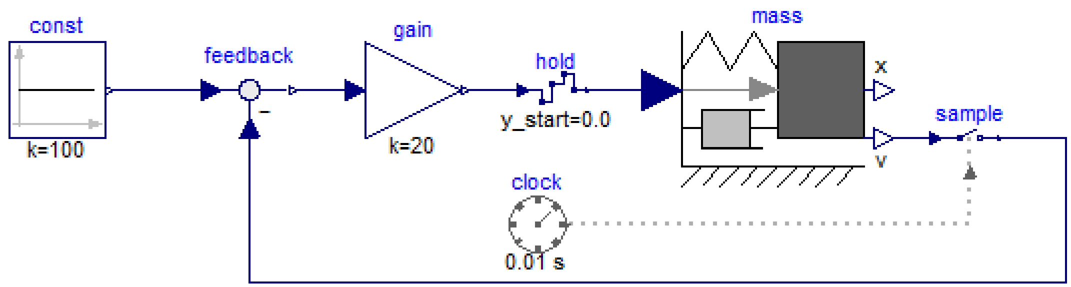
```
Clock c1 = Clock(1,10);             // period = 1/10
Clock c2 = superSample(c1,3);  // period = 1/30
Clock c3 = Clock(1,30);             // period = 1/30
```

Modelica_Synchronous library

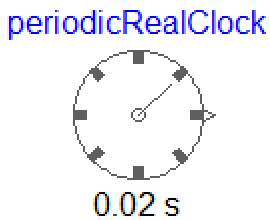
- Synchronous language elements of Modelica 3.3 are “low level”:

```
// speed sensor
vd = sample(v, Clock(0.01));

// P controller for speed
u = K*(vref-vd);

// force actuator
f = hold(u);
```

- Modelica_Synchronous library developed to access language elements in a convenient way graphically:

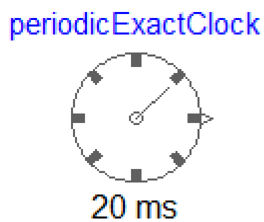


Blocks that generate clock signals

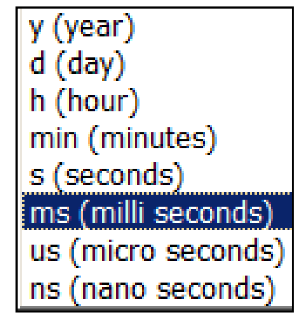


Generates a periodic clock with a Real period

```
  parameter Modelica.SIunits.Time period;
  ClockOutput y;
equation
  y = Clock(period);
```



Generates a periodic clock as an integer multiple of a resolution (defined by an enumeration).



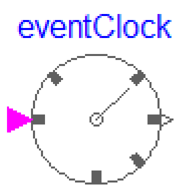
Code for 20 ms period:

```
y = superSample(Clock(20), 1000);
```

Clock with period 20 s

super-sample clock with 1000

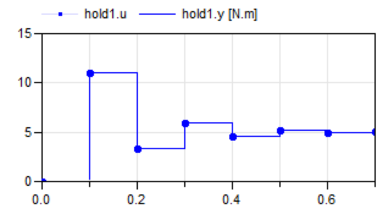
period = 20 / 1000 = 20 ms



Generates an event clock: The clock ticks whenever the continuous-time Boolean input changes from false to true.

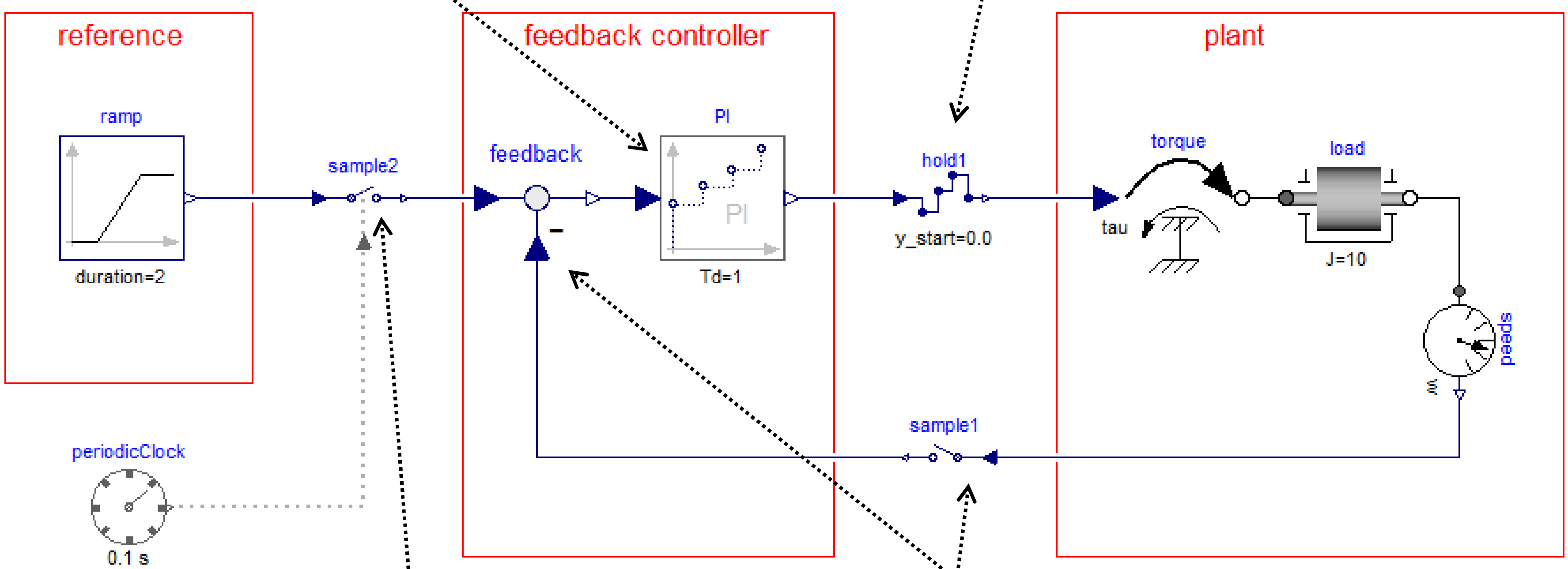
```
y = Clock(u);
```

Sample and Hold



Discrete-time PI controller

Holds a clocked signal and generates a continuous-time signal. Before the first clock tick, the continuous-time output y is set to parameter y_start `y = hold(u);`



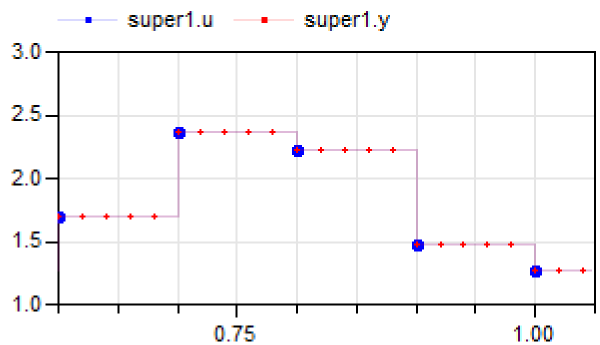
Samples a continuous-time signal and generates a clocked signal.

Purely algebraic block from Modelica.Blocks.Math

`y = sample(u, clock);`

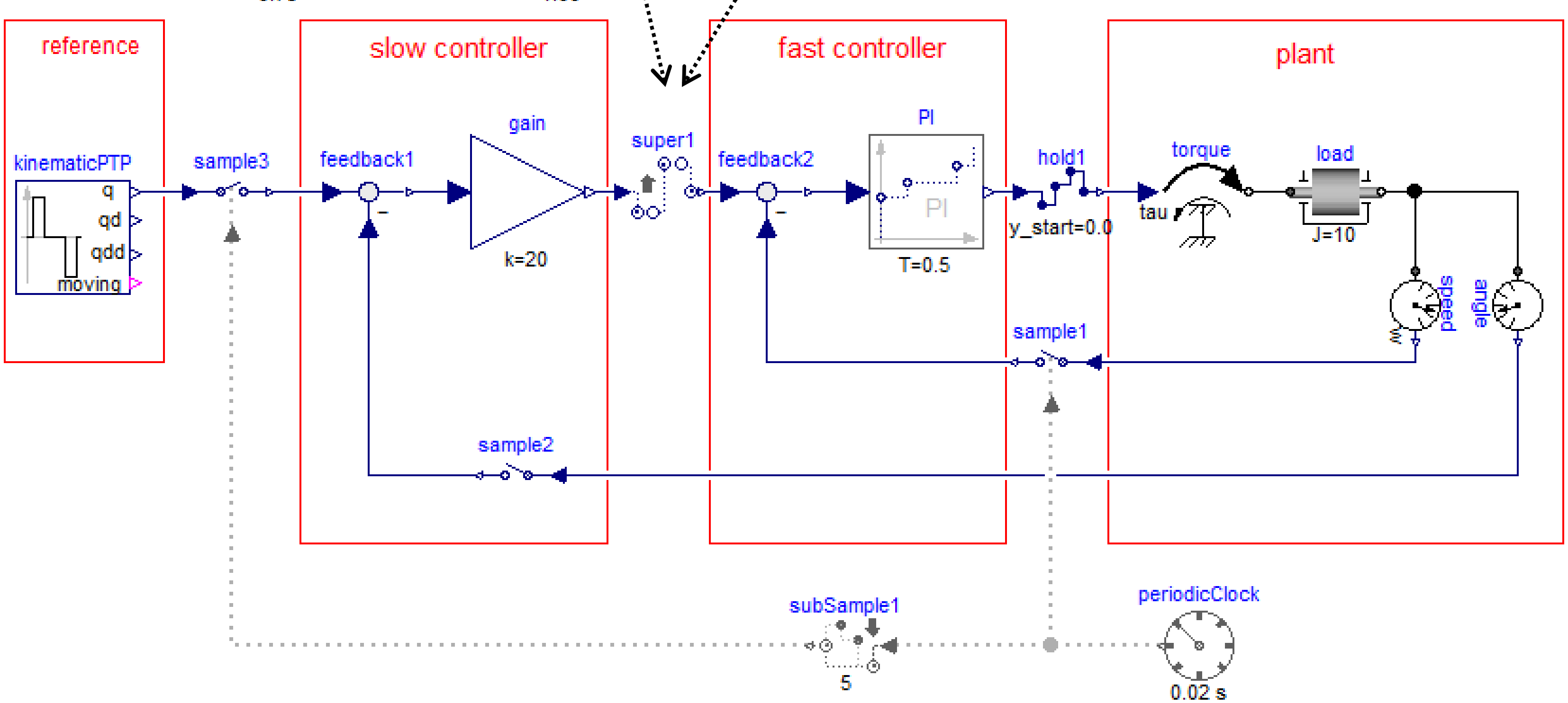
`y = sample(u);`

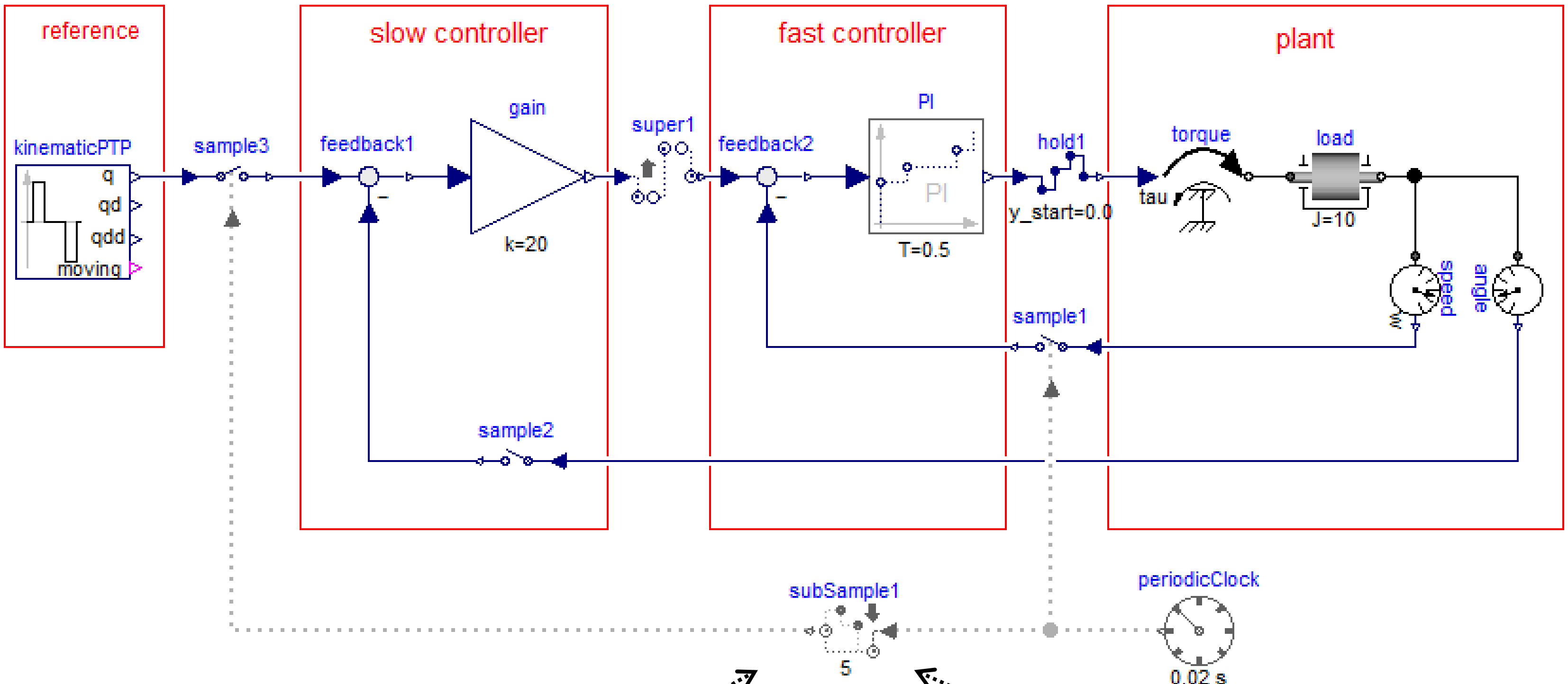
Sub- and Super-Sampling



Defines that the output signal is an integer factor faster as the input signal, using a “hold” semantics for the signal. By default, this factor is inferred. It can also be defined explicitly.

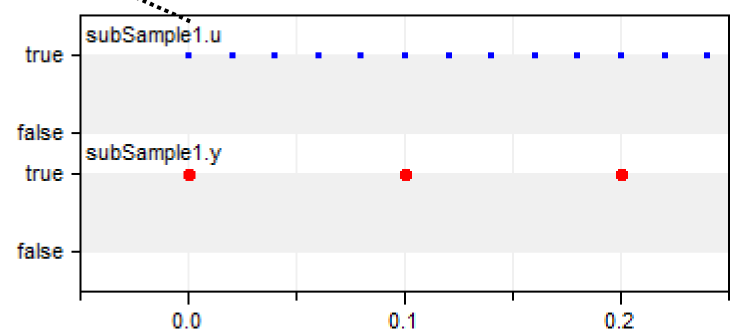
```
y = superSample(u);
```





Defines that the output signal is an integer factor slower as the input signal, picking every n-th value of the input.

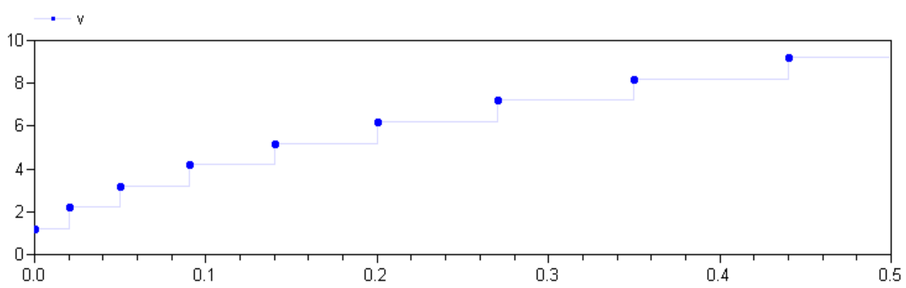
```
y = subSample(u,factor);
```



Varying Interval Clocks

- The first argument of Clock(ticks, resolution) may be time dependent
- Resolution must not be time dependent
- Allowing varying interval clocks
- Can be sub and super sampled and phased

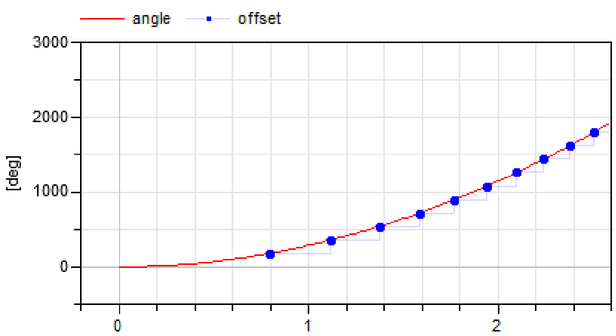
```
model VaryingClock
  Integer nextInterval(start=1);
  Clock c = Clock(nextInterval, 100);
  Real  v(start=0.2);
equation
  when c then
    nextInterval = previous(nextInterval) + 1;
    v = previous(v) + 1;
  end when;
end VaryingClock;
```



Boolean Clocks

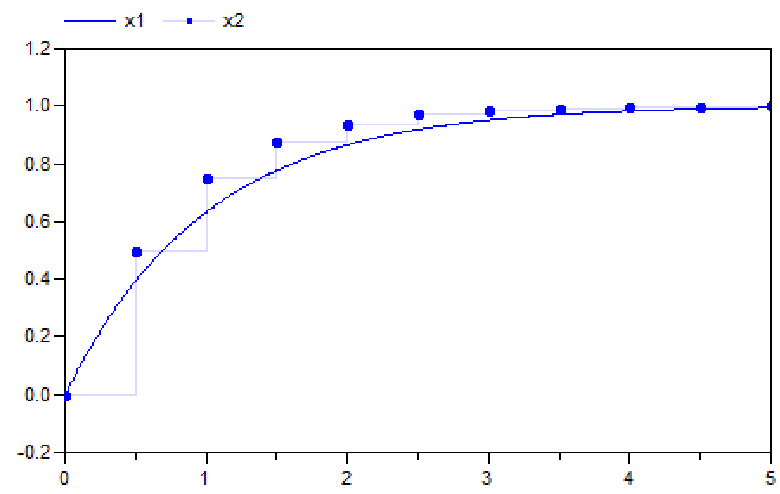
- Possible to define clocks that tick when a Boolean expression changes from false to true.
- Assume that a clock shall tick whenever the shaft of a drive train passes 180º.

```
model BooleanClock
  Modelica.SIunits.Angle angle(start=0,fixed=true);
  Modelica.SIunits.AngularVelocity w(start=0,fixed=true);
  Modelica.SIunits.Torque tau=10;
  parameter Modelica.SIunits.Inertia J=1;
  Modelica.SIunits.Angle offset;
equation
  w = der(angle);
  J*der(w) = tau;
  when Clock(angle >= hold(offset)+Modelica.Constants.pi) then
    offset = sample(angle);
  end when;
end BooleanClock;
```



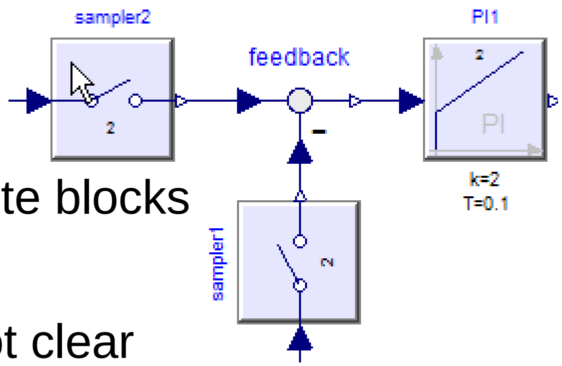
Discretized Continuous Time

- Possible to convert continuous-time partitions to discrete-time
- A powerful feature since in many cases it is no longer necessary to manually implement discrete-time components
- Build-up a inverse plant model or controller with continuous-time components and then sample the input signals and hold the output signals.
- And associate a solverMethod with the Clock.




```
model Discretized
  Real x1(start=0,fixed=true);
  Real x2(start=0,fixed=true);
equation
  der(x1) = -x1 + 1;

  der(x2) = -x2 + sample(1, Clock(Clock(0.5), solverMethod="ExplicitEuler"));
end Discretized;
```

Rationale for Clocked Semantics



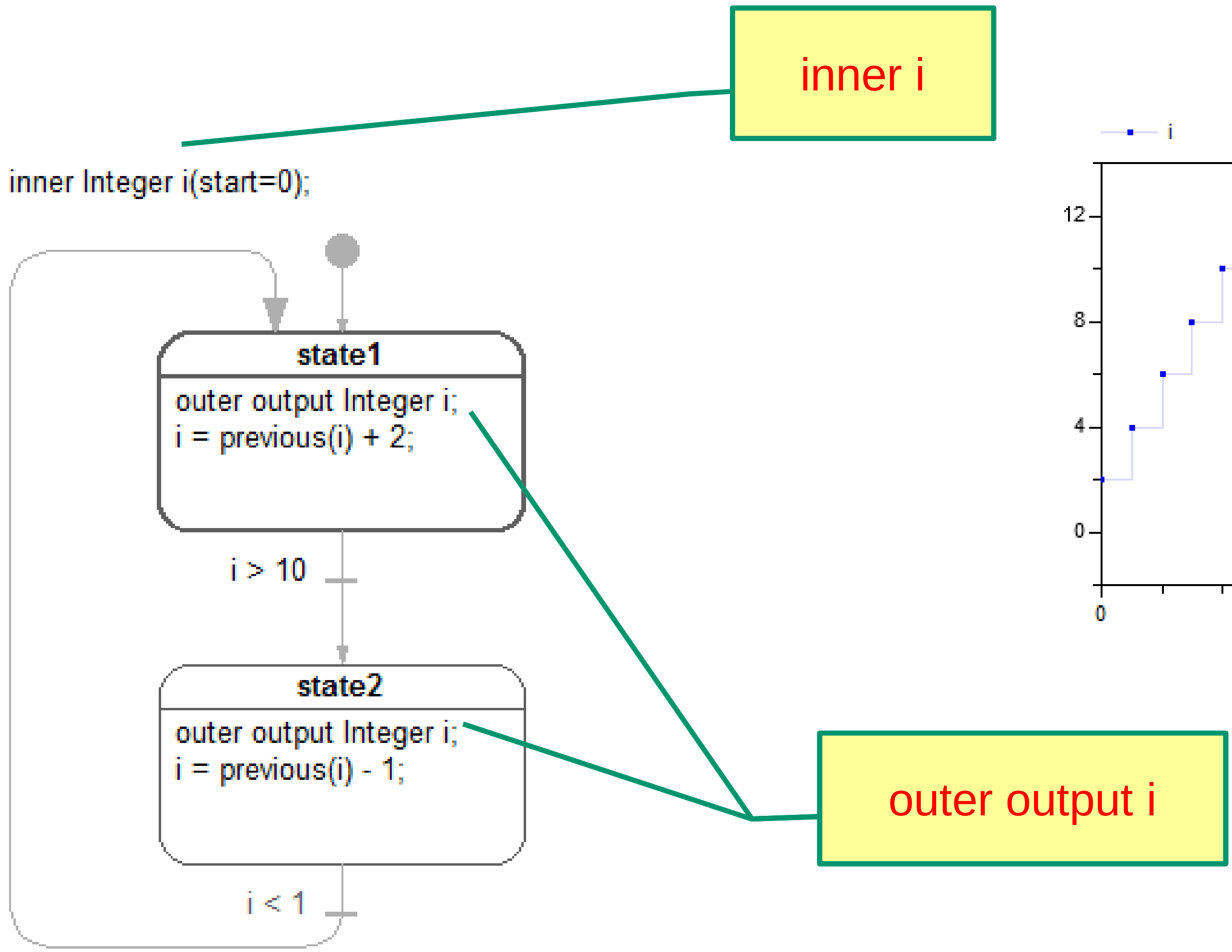
- Modelica 3.2
  - Discrete equations not general equations
  - Automatic hold on all variables
- Problems (Modelica_LinearSystems.Controller)
  - The sub-sample factors has to be given to all discrete blocks
  - Sampling errors cannot be detected by compiler
  - The boundaries between controller and plant are not clear (cf feedback above).
  - Unnecessary initial values have to be defined
  - Inverse models not supported in discrete systems
  - Efficiency degradation at event points.
- Modelica 3.3
  - Sampling period needs only to be given once (inferred)
  - Sampling errors detected since all variables in an equation must have the same clock
  - Only variables appearing in certain synchronous operators need start values

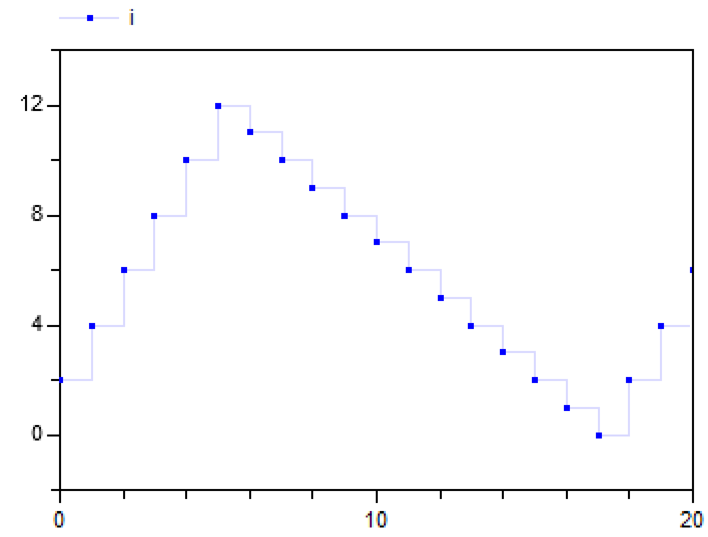
State Machines

- Modelica extended to allow modeling of control systems

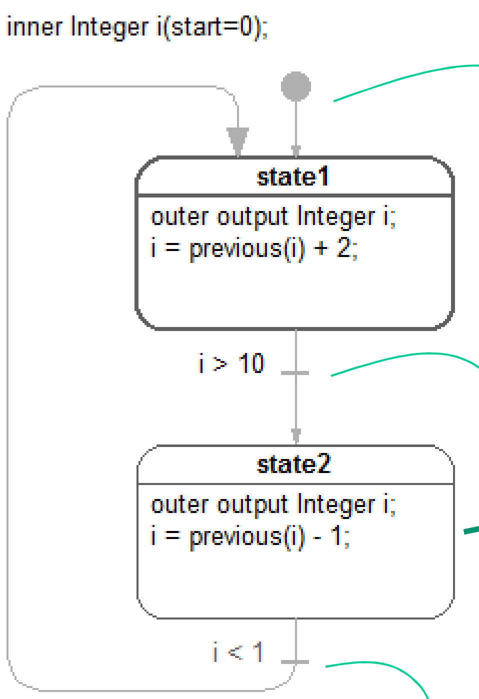
- Any block without continuous-time equations or algorithms can be a **state** of a state machine.
- Transitions between such blocks are represented by a new kind of connections associated with transition conditions.
- The complete semantics is described using only 13 Modelica equations.
- A cluster of block instances at the same hierarchical level which are coupled by **transition** equations constitutes a state machine.
- All parts of a state machine must have the same clock. *(We will work on removing this restriction ,allowing mixing clocks and allowing continuous equations, in future Modelica versions.)*
- One and only one instance in each state machine must be marked as initial by appearing in an **initialState** equation.

A Simple State Machine





A Simple State Machine – Modelica Text Representation




```
model StateMachine1
 inner Integer i(start=0);
 block State1
   outer output Integer i;
 equation
   i = previous(i) + 2;
 end State1;
 State1 state1;

 block State2
   outer output Integer i;
 equation
   i = previous(i) - 1;
 end State2;
 State2 state2;

equation
 initialState(state1);
 transition(state1, state2, i > 10, immediate=false);
 transition(state2, state1, i < 1, immediate=false);
end StateMachine1;
```

Merging Variable Definitions

- An **outer output** declaration means that the equations have access to the corresponding variable declared **inner**.
- Needed to maintain the single assignment rule.
- Multiple definitions of such outer variables in different mutually exclusive states of one state machine need to be merged.
- In each state, the outer output variables (v_j) are solved for ($expr_j$) and, for each such variable, a single definition is automatically formed:
- v := **if** activeState($state_1$) **then** $expr_1$
  elseif activeState($state_2$) **then** $expr_2$
  elseif … **else** last(v)

- **last**() is a special internal semantic operator returning its input. It is just used to mark for the sorting that the incidence of its argument should be ignored.
- A start value must be given to the variable if not assigned in the initial state.
- Such a newly created assignment equation might be merged on higher levels in nested state machines.

Defining a State machine

transition(from, to, condition, immediate, reset, synchronize, priority)

- This operator defines a transition from instance “from” to instance “to”. The “from” and “to” instances become states of a state machine.
- The transition fires when condition = true if immediate = true (this is called an “immediate transition”) or previous(condition) when immediate = false (this is called a “delayed transition”).
- If reset = true, the states of the target state are reinitialized, i.e. state machines are restarted in initial state and state variables are reset to their start values.
- If synchronize = true, the transition is disabled until all state machines within the from-state have reached the final states, i.e. states without outgoing transitions.

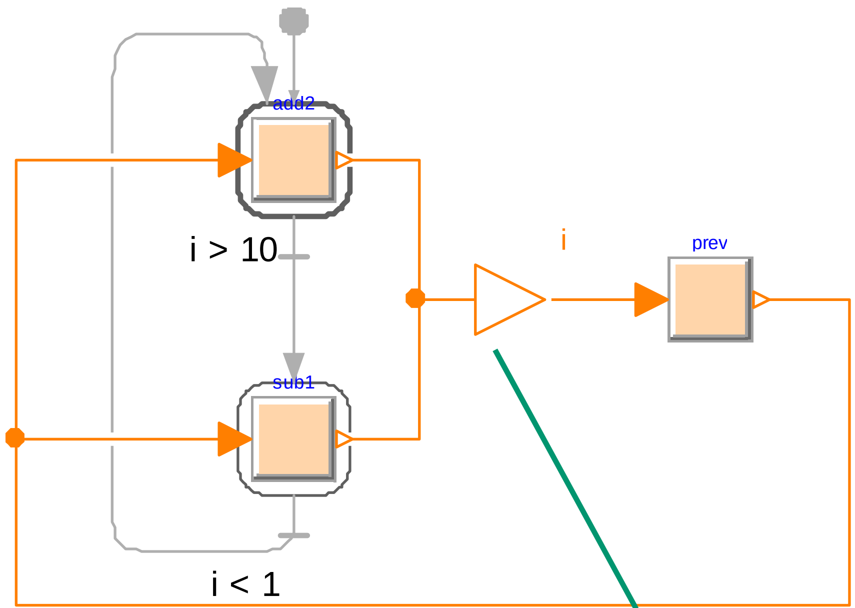
- “from” and “to” are block instances and “condition” is a Boolean expression.
- “immediate”, “reset”, and “synchronize” (optional) are of type Boolean, have parametric variability and a default of true, true, false respectively.
- “priority” (optional) is of type Integer, has parametric variability and a default of 1 (highest priority). Defines the priority of firing when several transitions could fire.

initialState(state)

- The argument “state” is the block instance that is defined to be the initial state of a state machine.

Conditional Data Flows

- Alternative to using **outer output** variables is to use conditional data flows.




```
block Increment
  extends Modelica.Blocks.Interfaces.PartialIntegerSISO;
  parameter Integer increment;
equation
  y = u + increment;
end Increment;
```

```
block Prev
  extends Modelica.Blocks.Interfaces.PartialIntegerSISO;
equation
  y = previous(u);
end Prev;
```

protected connector (node) i

Merge of Conditional Data Flows

- It is possible to connect several outputs to inputs if all the outputs come from states of the same state machine.

  $u_1 = u_2 = \ldots = y_1 = y_2 = \ldots$

  with u_i inputs and y_i outputs.
- Let variable v represent the signal flow and rewrite the equation above as a set of equations for u_i and a set of assignment equations for v:
- v := **if** activeState($state_1$) **then** y_1 **else** last(v);
  v := **if** activeState($state_2$) **then** y_2 **else** last(v);

  …
  $u_1 = v$
  $u_2 = v$

  …
- The merge of the definitions of v is then made as described previously:

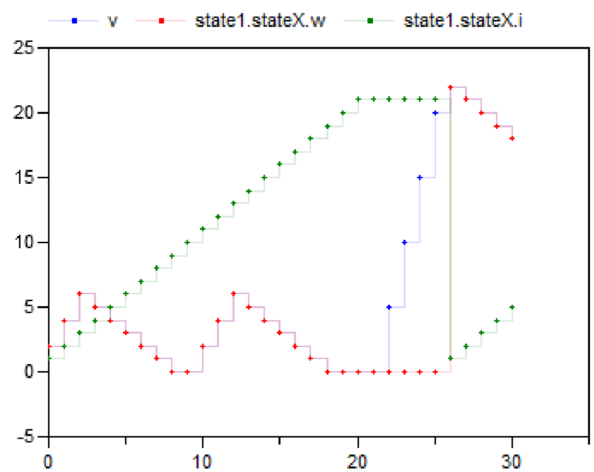
  v = **if** activeState($state_1$) **then** y_1

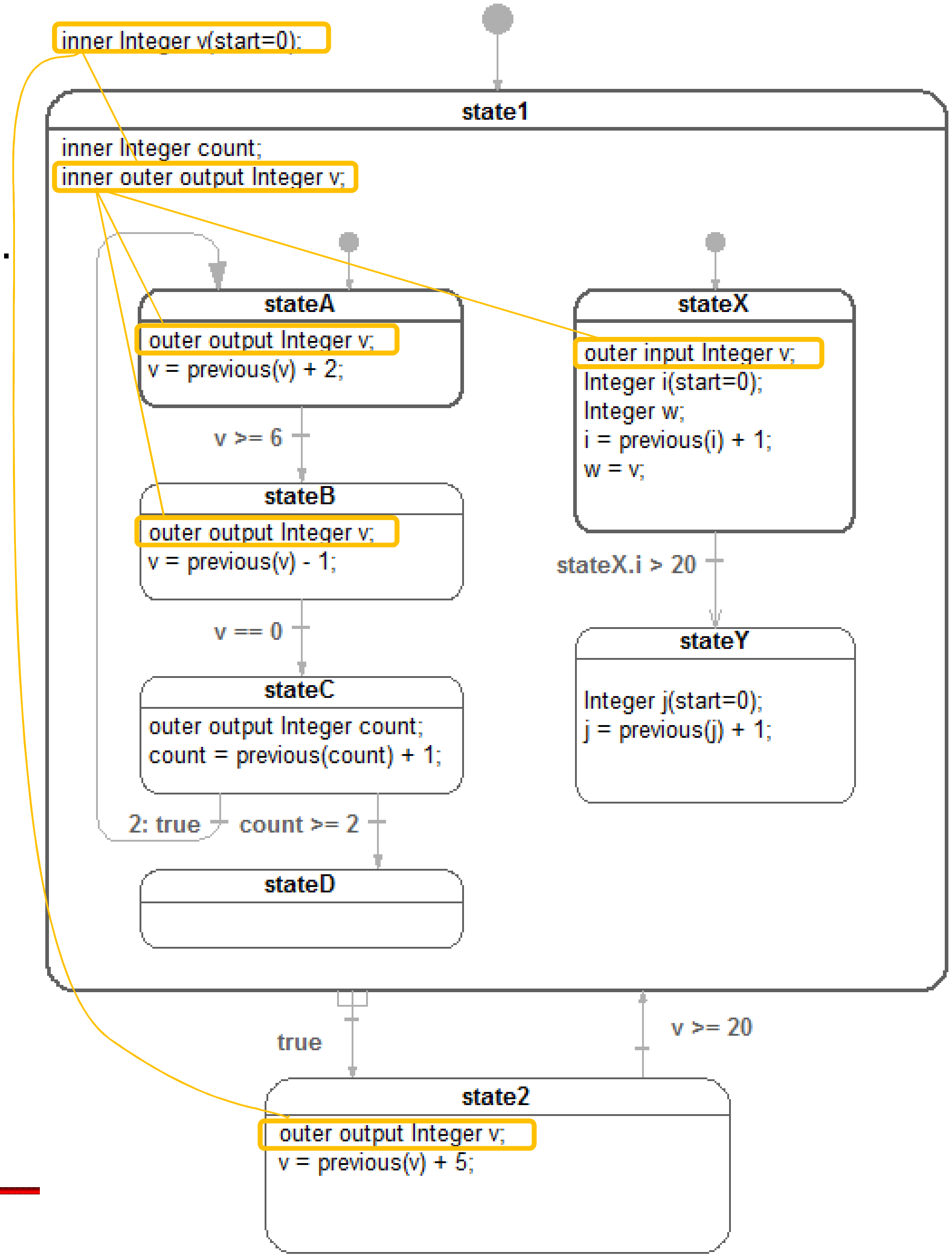
  elseif activeState($state_2$) **then** y_2
  elseif … **else** last(v)

  …

Hierarchical State Machine Example

- stateA declares v as ‘outer output’.
- state1 is on an intermediate level and declares v as ‘inner outer output’, i.e. matches lower level outer v by being inner and also matches higher level inner v by being outer.
- The top level declares v as inner and gives the start value.





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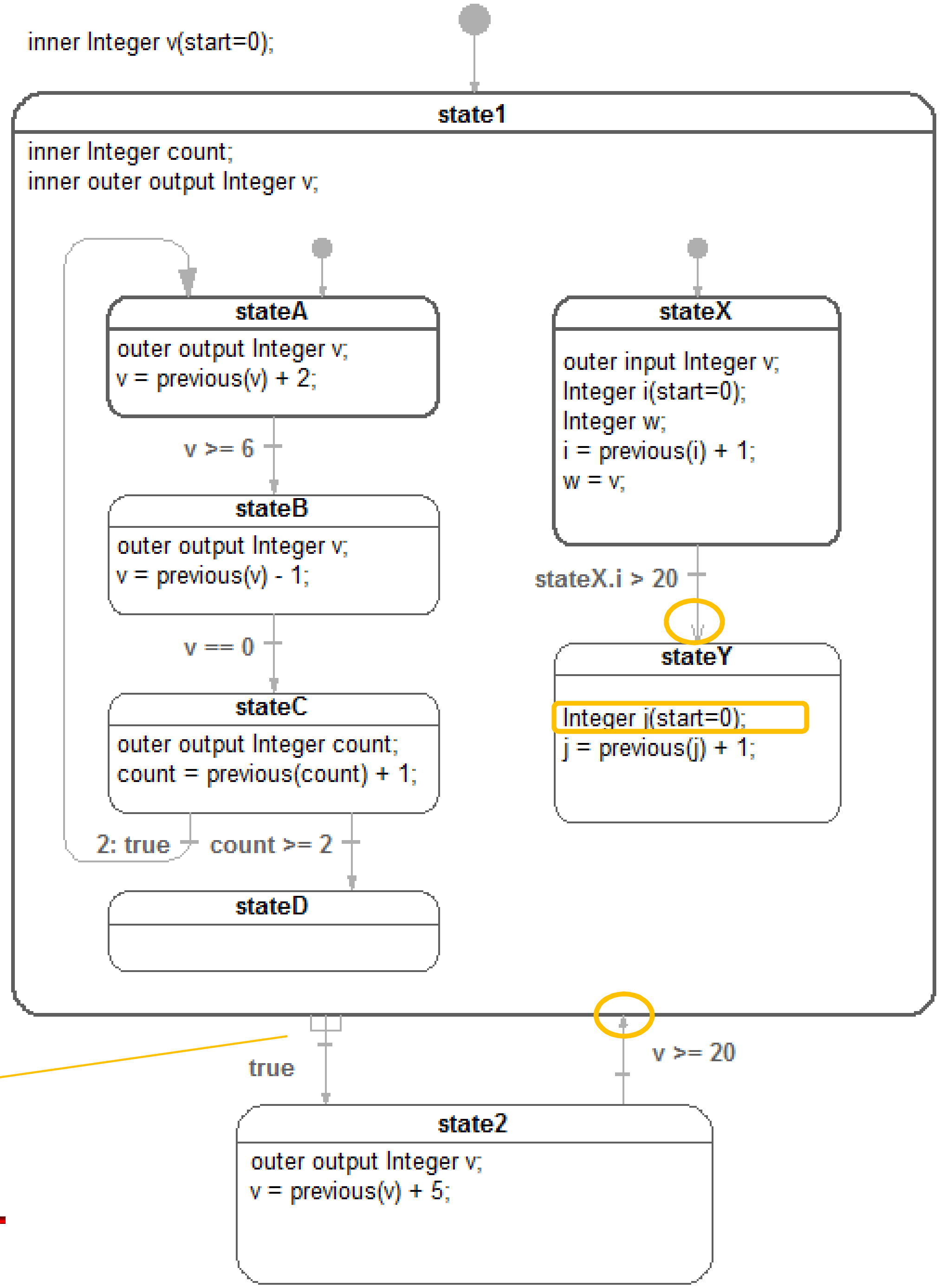
Reset and Synchronize

- count is defined with a start value in state1. It is reset when a reset transition (v>=20) is made to state1.
- stateY declares a local counter j. It is reset at start and as a consequence of the reset transition (v>=20) from state2 to state1.
- The reset of j is deferred until stateY is entered by transition (stateX.i>20) although this transition is not a reset transition.
- Synchronizing the exit from the two parallel state machines of state1 is done by using a synchronized transition.



State Machine Semantics

```
model StateMachineSemantics "Semantics of state machines"
  parameter Integer nStates;
  parameter Transition t[:]  "Array of transition data sorted in priority";
  input Boolean c[size(t,1)]  "Transition conditions sorted in priority";

  Boolean active "true if the state machine is active";
  Boolean reset "true when the state machine should be reset";
  Integer selectedState = if reset then 1 else previous(nextState);
  Boolean selectedReset = if reset then true else previous(nextReset);

// For strong (immediate) and weak (delayed) transitions
  Integer immediate = max(if (if t[i].immediate and t[i].from == selectedState then c[i] else false)
     then i else 0 for i in 1:size(t,1));
  Integer delayed = max(if (if not t[i].immediate and t[i].from == nextState then c[i] else false)
     then i else 0 for i in 1:size(t,1));
  Integer fired = max(previous(delayed), immediate);
  output Integer activeState = if reset then 1  elseif fired > 0 then t[fired].to else selectedState;
  output Boolean activeReset = if reset then true  elseif fired > 0 then t[fired].reset else selectedReset;
```

State Machine Semantics II

```
// Update states
  Integer nextState = if active then activeState else previous(nextState);
  Boolean nextReset = if active then false else previous(nextReset);

// Delayed resetting of individual states
  output Boolean activeResetStates[nStates] =
     {if reset then true else previous(nextResetStates[i]) for i in 1:nStates};
  Boolean nextResetStates[nStates] = if active then {if selectedState == i then false else activeResetStates[i]
     for i in 1:nStates} else previous(nextResetStates);
  Boolean finalStates[nStates] = {max(if t[j].from == i then 1 else 0 for j in 1:size(t,1)) == 0 for i in 1:nStates};
  Boolean stateMachineInFinalState = finalStates[activeState];
end StateMachineSemantics;
```

Comparison to Other State Machine Formalisms

- State machines needed to be introduced in Modelica to enable modeling of complete systems. Several attempts have been made:
  - (*Mosterman et. al. 1998)*, defines state machines in an object-oriented way with Boolean equations.
  - A more powerful state machine formalism was introduced in StateGraph (*Otter et. al. 2005*).
  - A prototype mode automata formalism was implemented (*Malmheden et. al. 2008*) using a built-in concept of modes.
  - Certain problems of potentially unsafe models in StateGraph were removed in the StateGraph2 library (*Otter et. al. 2009*).
  - These efforts showed that state machine support must be natively supported in the language.
- The presented state machines of Modelica 3.3 have a similar modeling power as Statecharts (*Harel, 1987*) and State Machine Diagrams of SysML (*Friedenthal 2008*).

Comparison to Other State Machine Formalisms II

- The semantics of the state machines defined in this paper is inspired by mode automata (*Maraninchi 2002*) and basically the same as Lucid Synchrone 3.0 (*Pouzet 2006*), or its clone LCM (Logical Control Module) (*Gaucher et.al. 2009*).
- Some minor properties are different compared to Lucid Synchrone 3.0, in particular regarding transition conditions.
  - Lucid Synchrone has two kinds of transitions: namely “strong” and “weak”.
  - Strong transitions are executed before the actions of a state are evaluated while weak transitions are executed after.
  - This can lead to surprising behavior, because the actions of a state are skipped if it is activated by a weak transition and exited by a true strong transition.
  - For this reason, the state machines in Modelica use “immediate” (= the same as “strong”) and “delayed” transitions. Delayed transitions are “immediate” transitions where the condition is automatically delayed with an implicit **previous**(...).

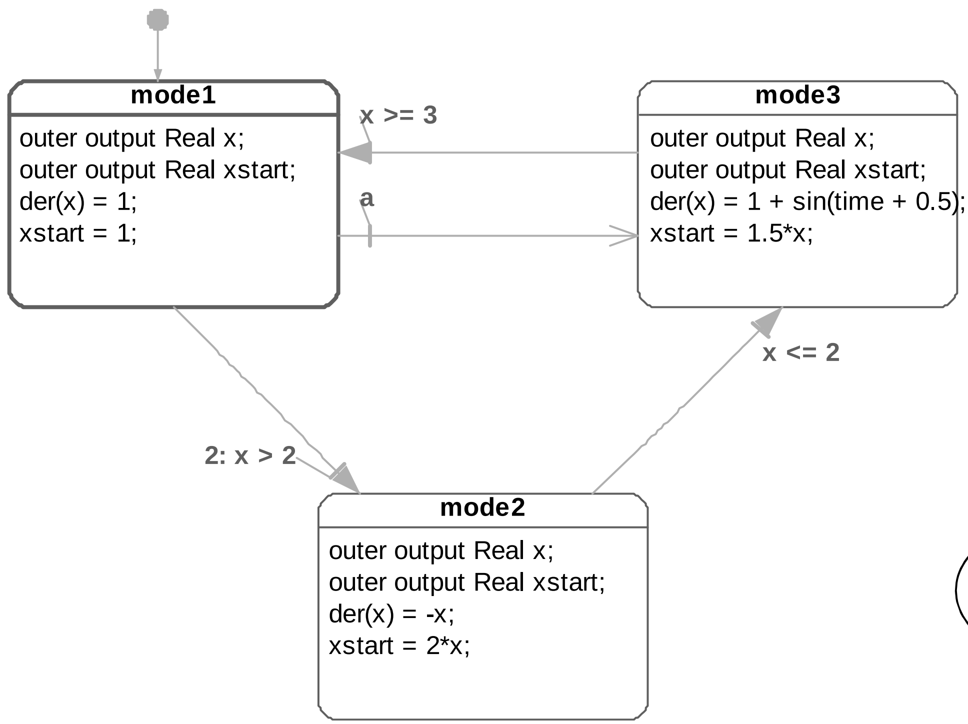
Comparison to Other State Machine Formalisms III

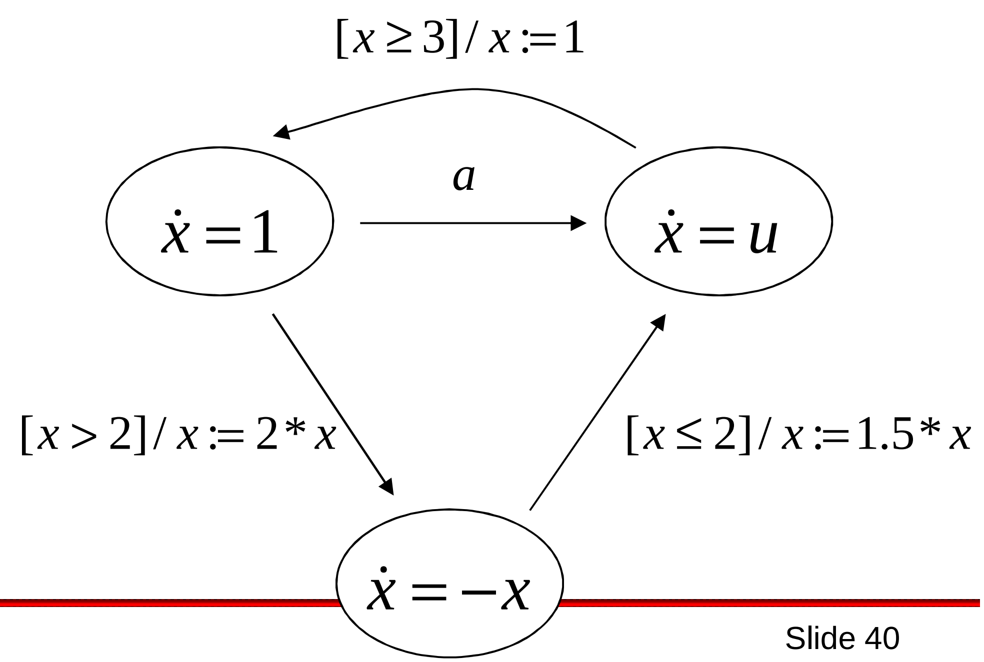
- Safety critical control software in aircrafts is often defined with such kind of state machines, such as using the Scade 6 Tool Suite from Esterel Technologies (*Dormoy 2008*) that provides a similar formalism as Lucid Synchrone,
  - With differences such as the ability to associate actions to transitions in addition to states.
  - Scade also provides synchronize semantics by means of synchronization transitions between several parallel sub-state machines being in states which have been declared final.
- Stateflow (*Mathworks 2012*), while being very expressive, suffers from “numerous, complex and often overlapping features lacking any formal definition”, as reported by (*Hamon, et.al, 2004)*.
- The Modelica approach has the important property that at one clock tick, there is only one assignment to every variable.
- Modelica, Lucid Synchrone, LCM and Scade 6 all have the property that data flow and state machines can be mutually hierarchically structured, i.e. that, for example a state of a state machine can contain a block diagram in which the blocks might contain state machines.

Hybrid Automata

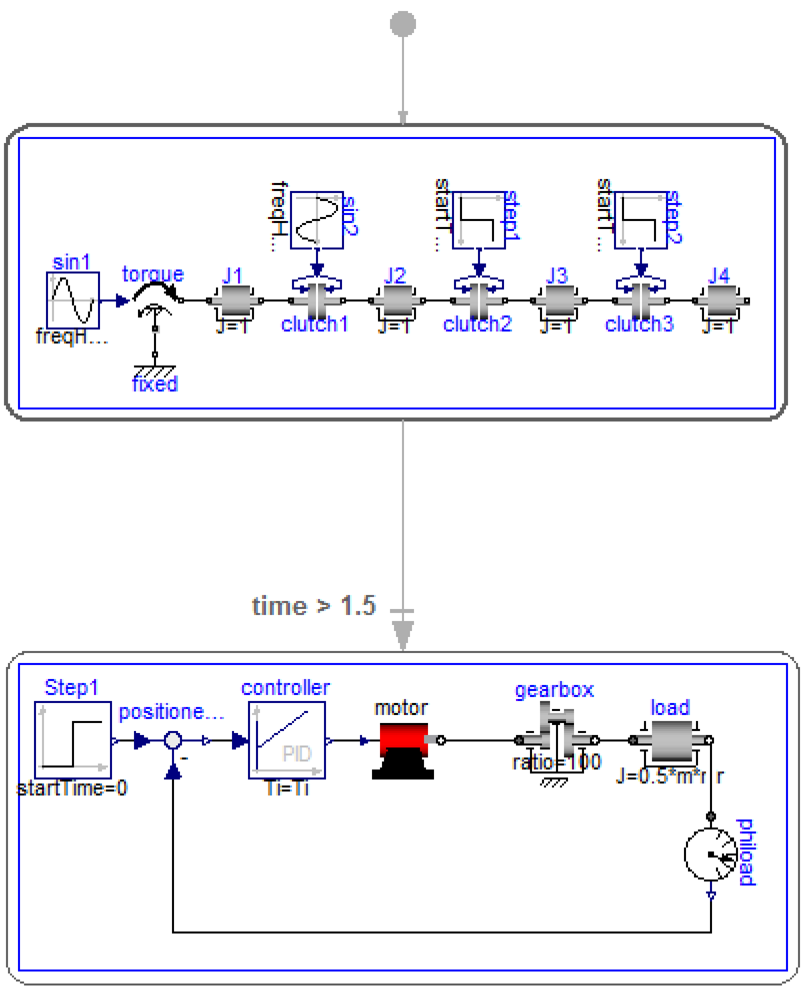
Hybrid Automata with Modelica 3.3+ (prototype)

```
inner Real xstart(start=1, fixed=true);
inner Real x(start=xstart, fixed=true);
Boolean t3=time > 2.5;
Boolean a=edge(t3);
```





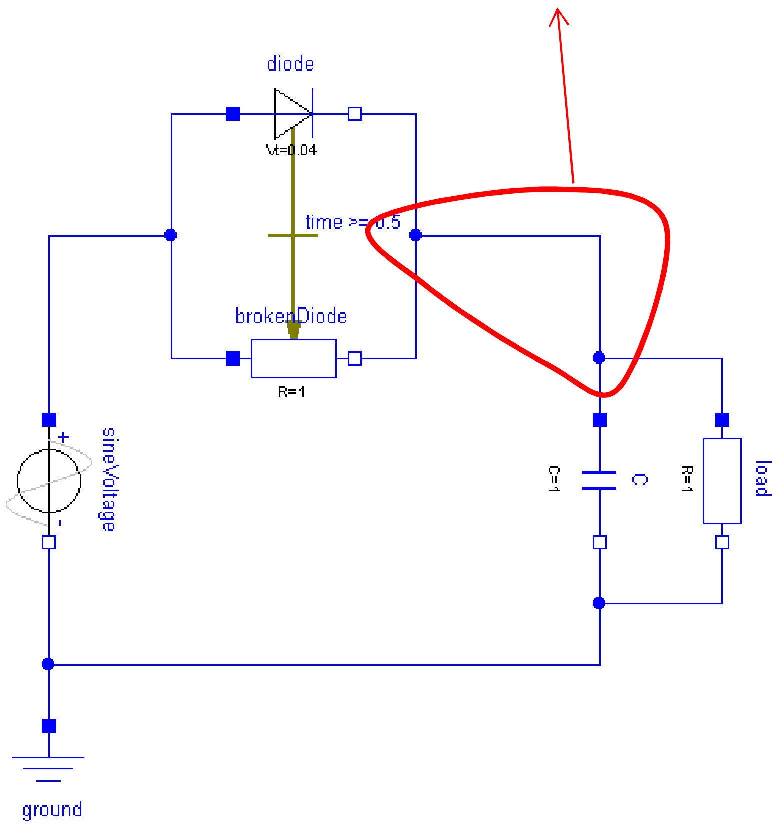
Acausal Models in States – Modelica 3.3+

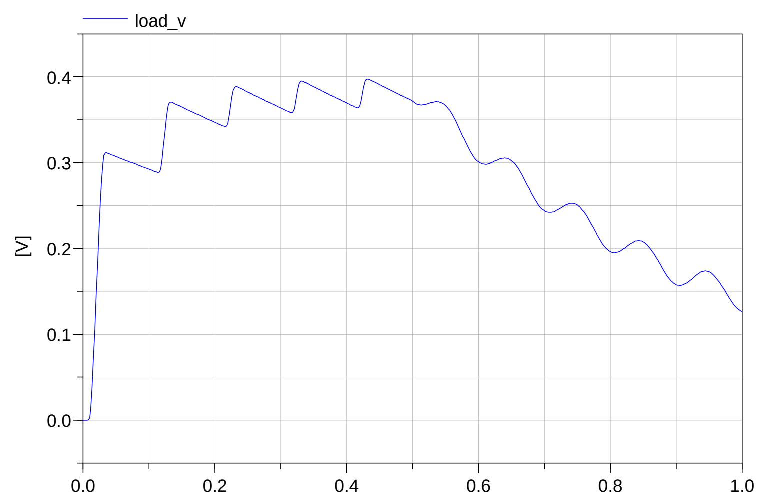


- The equations of each state is guarded by the activity condition
- Should time variable be stopped when not active?
- Should time be reset locally in state by a reset transition?
- Special Boolean operator exception() to detect a problem in one model and transition to another model

Multiple Acasual Connections

- // C_p_i+brokenDiode_n_i+diode_n_i+load_p_i = 0.0;
- Replaced by:
- C_p_i +
  (if activeState(brokenDiode) then brokenDiode_n_i else 0) +
  (if activeState(diode) then diode_n_i else 0) +
  load_p_i = 0.0;





Conclusions

- We have introduced synchronous features in Modelica 3.3.
- For a discrete-time variable, its clock is associated with the variable type and inferencing is supported.
- Special operators have to be used to convert between clocks.
- This gives an additional safety since correct synchronization is guaranteed by the compiler.

- We have described how state machines can be modeled in Modelica 3.3.
- Instances of blocks connected by transitions with one such block marked as an initial state constitute a state machine.
- Hierarchical state machines can be defined with reset or resume semantics, when re-entering a previously executed state.
- Parallel sub-state machines can be synchronized when they reached their final states.
- Special merge semantics have been defined for multiple outer output definitions in mutually exclusive states as well as conditional data flows.