Hybrid Systems Theory
presented by Tom Henzinger

1. Robust hybrid systems
2. Stochastic hybrid systems
3. Compositional hybrid systems
4. Computational hybrid systems

Chess

NSF

Program Review
May 10, 2004
Berkeley, CA

VANDERBILT UNIVERSITY
INSTITUTE FOR SOFTWARE INTEGRATED SYSTEMS

UC Berkeley: Chess
Vanderbilt University: ISIS
University of Memphis: MSI

Foundations of Hybrid and Embedded Software Systems

A Formal Foundation for Embedded Systems

needs to combine

Computation + Physicality

Theories of
-composition & hierarchy
-computability & complexity

→ R

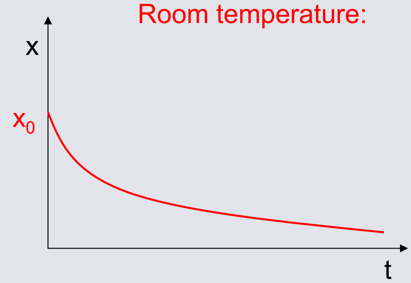
Theories of
-robustness & approximation
-probabilities & discounting

B ←

Continuous Dynamical Systems

State space: R^n
Dynamics: initial condition + differential equations

Room temperature: $x(0) = x_0$
$x'(t) = -K \cdot x(t)$

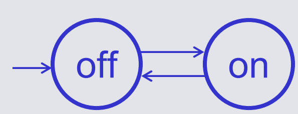


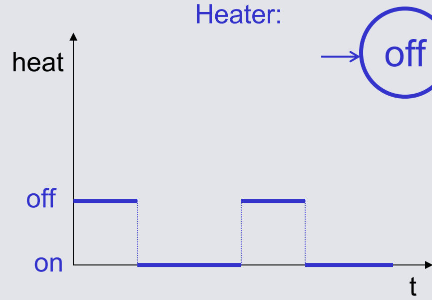
Analytic complexity.

Discrete Transition Systems

State space: B^m
Dynamics: initial condition + transition relation

Heater:



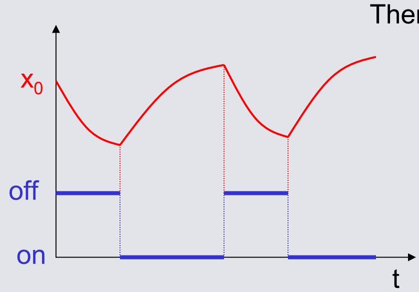


Combinatorial complexity.

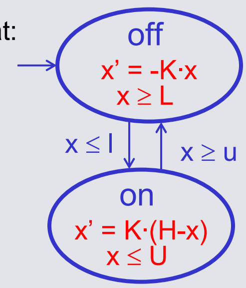
Hybrid Automata

State space: $B^m \times R^n$

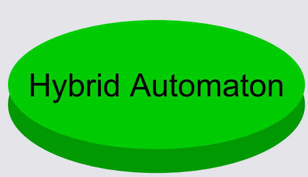
Dynamics: initial condition + transition relation + differential equations

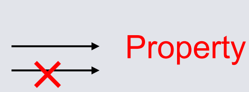


Thermostat:



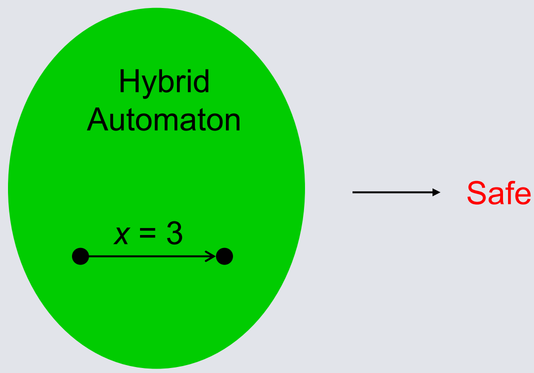
The Robustness Issue



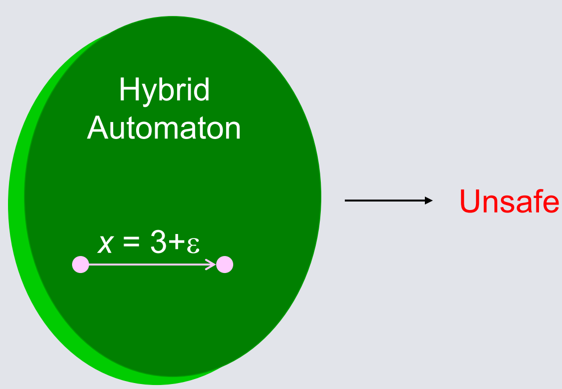


slightly perturbed automaton

The Robustness Issue



The Robustness Issue



Towards Robust Hybrid Automata

value(Model,Property): States $\rightarrow$ B

value(Model,Property): States $\rightarrow$ R

Towards Robust Hybrid Automata

value(Model,Property): States $\rightarrow$ B

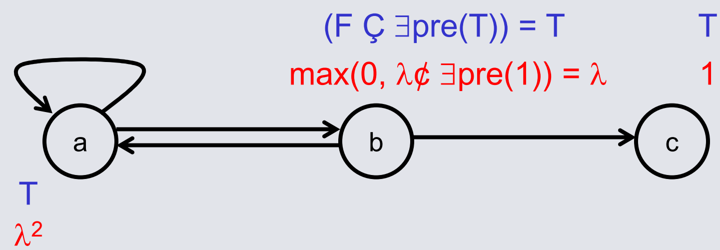
value(m, $\Diamond T$) = $(\mu X)\ (T \vee \text{pre}(X))$

discountedValue(Model,Property): States $\rightarrow$ R

discountedValue(m, $\Diamond T$) = $(\mu X)\ \max(T, \lambda \cdot \text{pre}(X))$

discount factor $0<\lambda<1$

Reachability (coSafety)



$\exists \Diamond c$ … undiscounted property

$\exists \Diamond_{\lambda} c$ … discounted property

Main Result (so far, only for discrete systems)

Robustness Theorem [de Alfaro, Henzinger, Majumdar]:

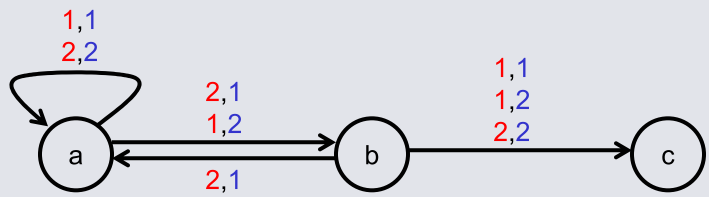
If discountedBisimilarity(m_1,m_2) > 1 - ε,
then |discountedValue(m_1,p) - discountedValue(m_2,p)| < $f(\varepsilon)$.

Further Advantages of Discounting:

-approximability because of geometric convergence (avoids non-termination of verification algorithms)

-applies also to probabilistic systems and to games (enables reasoning under uncertainty, and control)

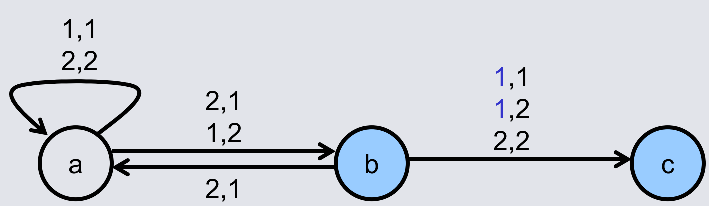
Concurrent Game



player "left"
player "right"

-for modeling component-based systems (“interfaces”)
-for strategy synthesis (“control”)

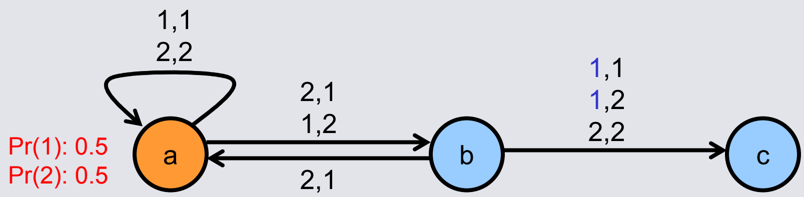
Concurrent Game



$\exists_{left}\ \forall_{right}\ \diamond\ c$ … player "left" has a deterministic strategy to reach c

$$(\mu X)\ (T \vee \exists_{left}\forall_{right}\mathrm{pre}(X))$$

Concurrent Game



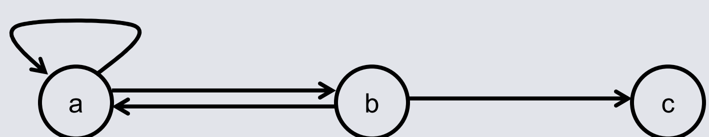
$\exists_{left}\ \forall_{right}\ \Diamond\ c$ ... player "left" has a deterministic strategy to reach c

$\mathit{\exists}_{left}\ \mathit{\forall}_{right}\ \Diamond\ c$ ... player "left" has a randomized strategy to reach c

$$(\mu X)\ (T \vee \mathit{\exists}_{left}\ \mathit{\forall}_{right} \mathrm{pre}(X))$$

Stochastic Game

Probability with which player "left" can reach c ?



State a:

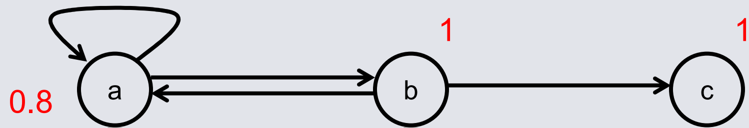
left \ right	1	2
1	a: 0.6 b: 0.4	a: 0.5 b: 0.5
2	a: 0.1 b: 0.9	a: 0.2 b: 0.8

State b:

left \ right	1	2
1	a: 0.0 c: 1.0	a: 0.0 c: 1.0
2	a: 0.7 b: 0.3	a: 0.0 b: 1.0

Stochastic Game

Probability with which player "left" can reach c ?



right / left	1	2
1	a: 0.6 b: 0.4	a: 0.5 b: 0.5
2	a: 0.1 b: 0.9	a: 0.2 b: 0.8

right / left	1	2
1	a: 0.0 c: 1.0	a: 0.0 c: 1.0
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$(\mu X)\ \max(T,\ \exists_{left}\ \forall_{right} pre(X))$

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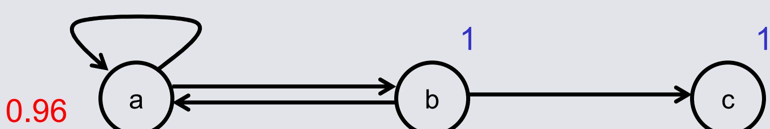

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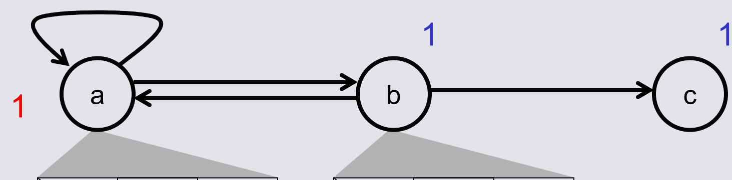

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$(\mu X)\ \max(T, \exists_{left} \forall_{right} pre(X))$

Hybrid Systems Theory

1 Robust hybrid systems

2 Stochastic hybrid systems

Theory of discounting.
Timed and stochastic games and optimal control.

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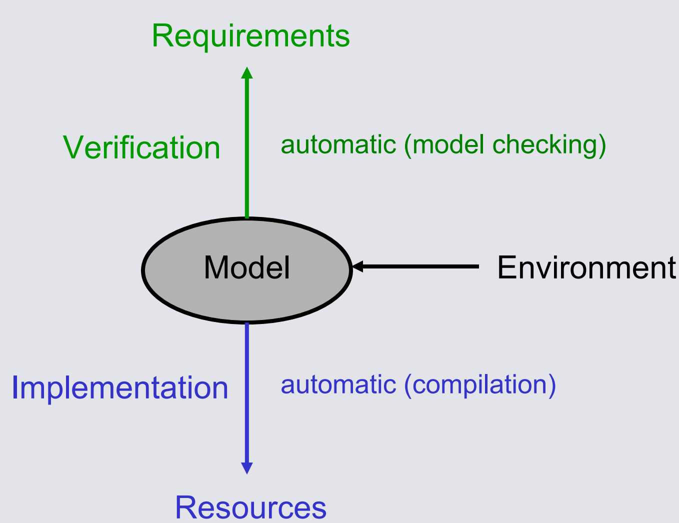
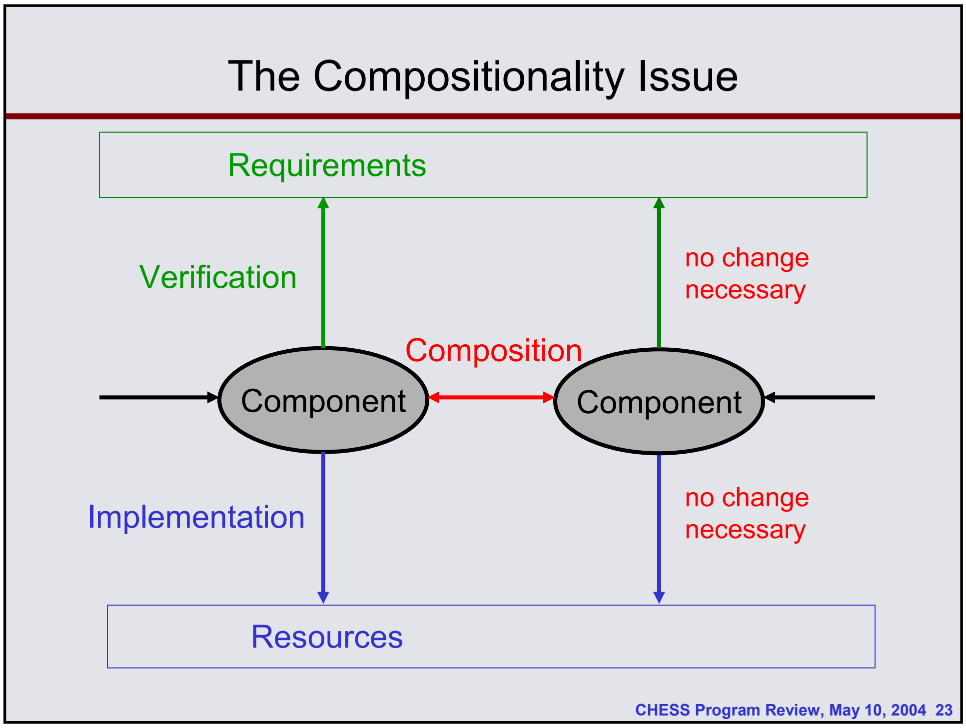
Agent algebras and interface theories.
Hybrid systems simulation (Ptolemy II, GME).
Hybrid systems verification (polyhedral & ellipsoidal methods).
Code generation from hybrid systems (Ptolemy II, Giotto).

The Compositionality Issue





The Compositionality Issue
Requirements
Verification
no change
necessary
Composition
Component
Component
Implementation
no change
necessary
Resources
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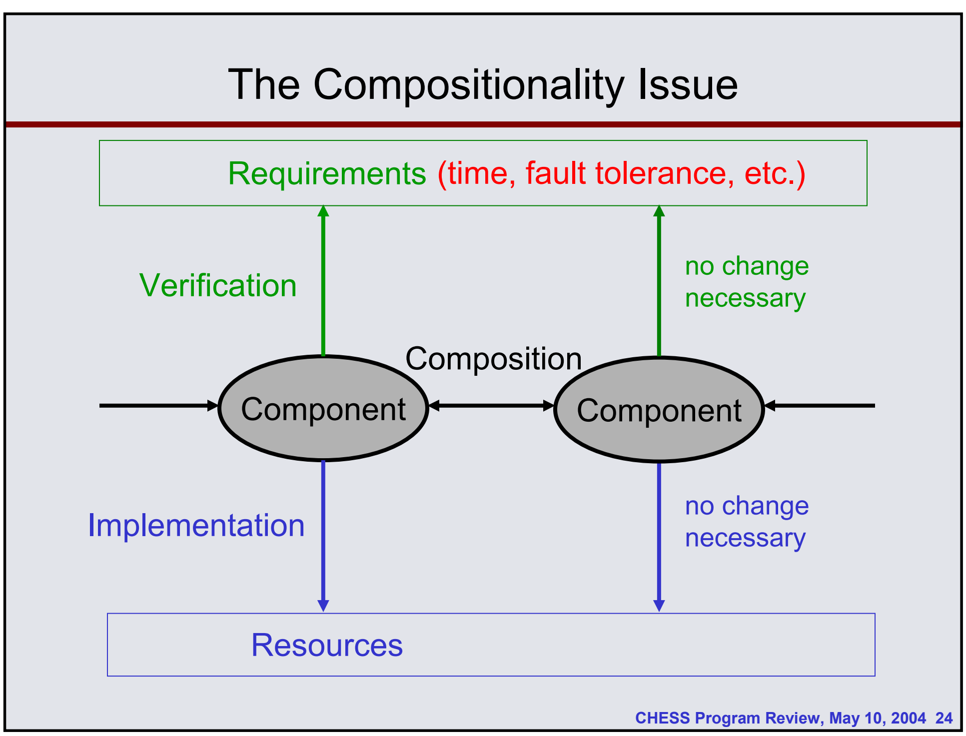

The Compositionality Issue
Requirements (time, fault tolerance, etc.)
Verification
no change
necessary
Composition
Component
Component
Implementation
no change
necessary
Resources
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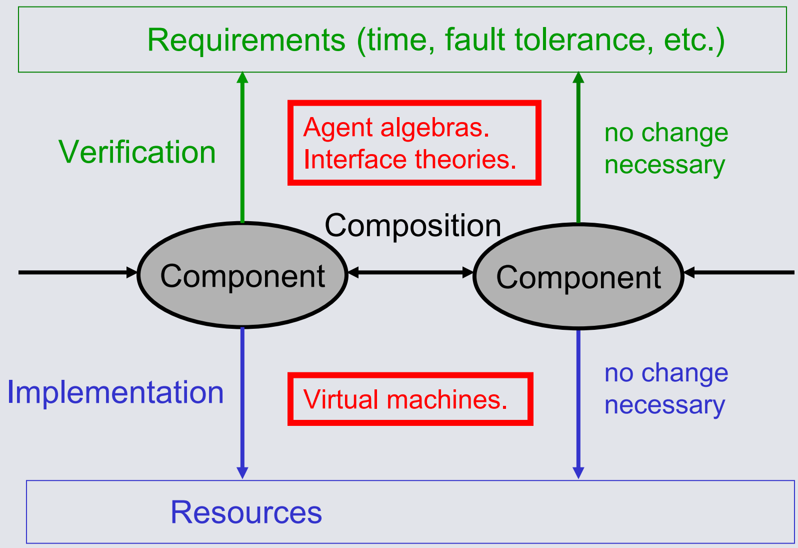
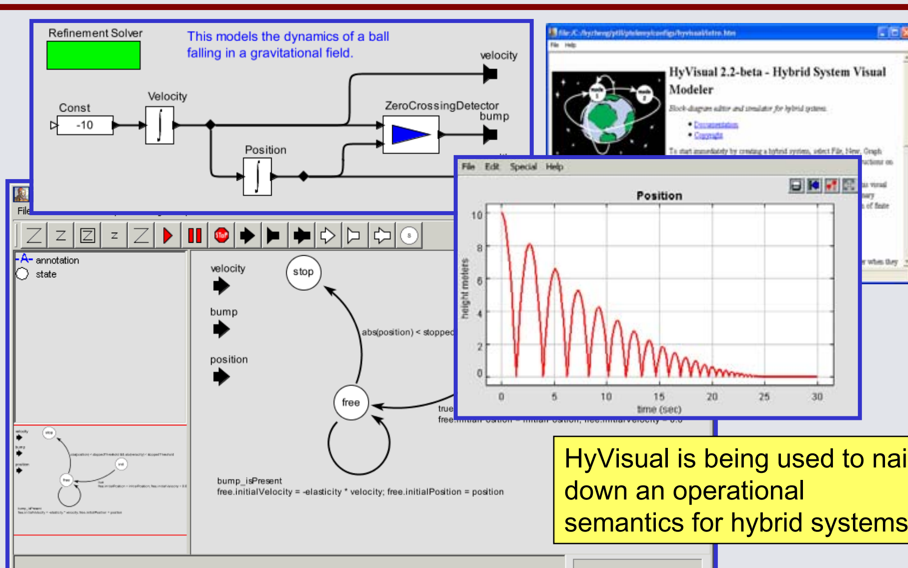
The Compositionality Issue

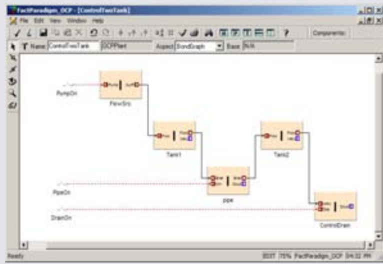


Compositional Hybrid Systems Modeling and Simulation I: HyVisual (based on Ptolemy II)



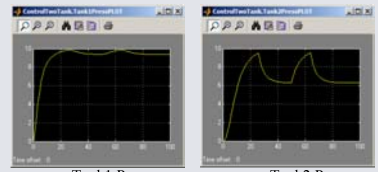
HyVisual is being used to nail down an operational semantics for hybrid systems.

Compositional Hybrid Systems Modeling and Simulation II: Generating SIMULINK models from Hybrid Bond Graphs



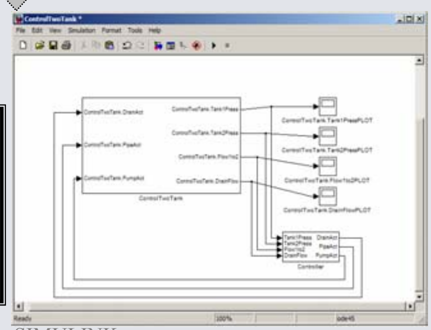
Two Tank Example

GME



Tank1 Pressure

Tank2 Pressure



SIMULINK

THE SIMULINK ENVIROMENT

- Using Simulink blocks and subsystems we can create a block diagram that corresponds to the Hybrid Bond Graph in GME.
- Simulink provides variable step solvers for the simulation of ODEs, with error control and zero crossing detection.

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Continuous System Verification: Problem

Given

- control system $\dot{x}(t) \in Ax(t) + BU$
- control set U
- target set M

Consider decision problems

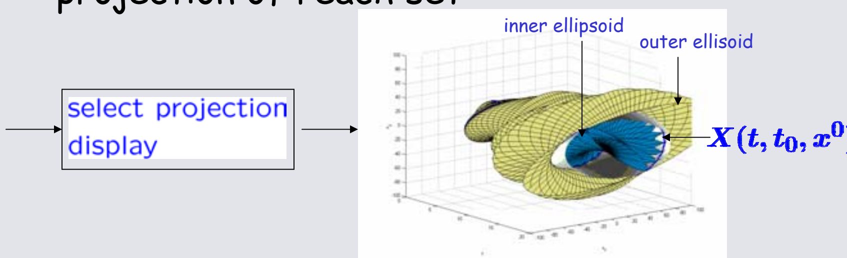
$$X(t_1, t_0, x^0) \subset M?$$
$$X(t_1, t_0, x^0) \cap M = \emptyset?$$
$$X(t_1, t_0, x^0) \cap M \neq \emptyset?$$

Continuous System Verification: Approach

1. Express decision problems as optimization problems
2. Derive Hamilton-Jacobi-Bellman (HJB) partial differential equation of value functions
3. Obtain support functions of $X(t, t_0, x^0)$ using convex analysis
4. Obtain tight outer and inner ellipsoidal approximations to $X(t, t_0, x^0)$

Continuous System Verification: Implementation

1. Matlab 'toolbox' to calculate ellipsoidal approximations and display any 2-d projection of reach set



2. Solve control problem: find control to steer $(t_0, x^0) \longrightarrow (t_1, x^1)$

From Continuous to Hybrid System Verification

1. Find states that can be reached despite disturbances $\dot{x}(t) \in Ax(t) + BU + CV$ in which V is set of disturbances

2. Find reach set of hybrid systems

$$\dot{x}(t) \in A_i x(t) + B_i U$$

in which i-th system is triggered by 'guard'

$$\langle c_i, x \rangle = d_i$$

(future work)

Related In-Depth Talks

3:50 p.m. Compositional Hybrid Systems:

Model Transformations on Hybrid Models
(Aditya Agrawal)

4:10 p.m. Stochastic Hybrid Systems:

Hybrid Systems in Systems Biology
(Wei Chung Wu, Jianghai Hu, Shankar Sastry)

4:30 p.m. Computational Hybrid Systems:

Computational Methods for Analyzing and
Controlling Hybrid Systems (Claire Tomlin)

Related Posters

Compositional Hybrid Systems:

Rich Interface Theories (Arindam Chakrabarti)
Compositional Metamodeling (Matthew Emerson)

Stochastic Hybrid Systems:

Stochastic Hybrid Systems (Alessandro Abate)

Computational Hybrid Systems:

Zeno Behavior in Hybrid Systems (Aaron Ames)
Computation of Reach Sets (Alex Kurzhansky)