Computational Methods for Analyzing and Controlling Hybrid Systems

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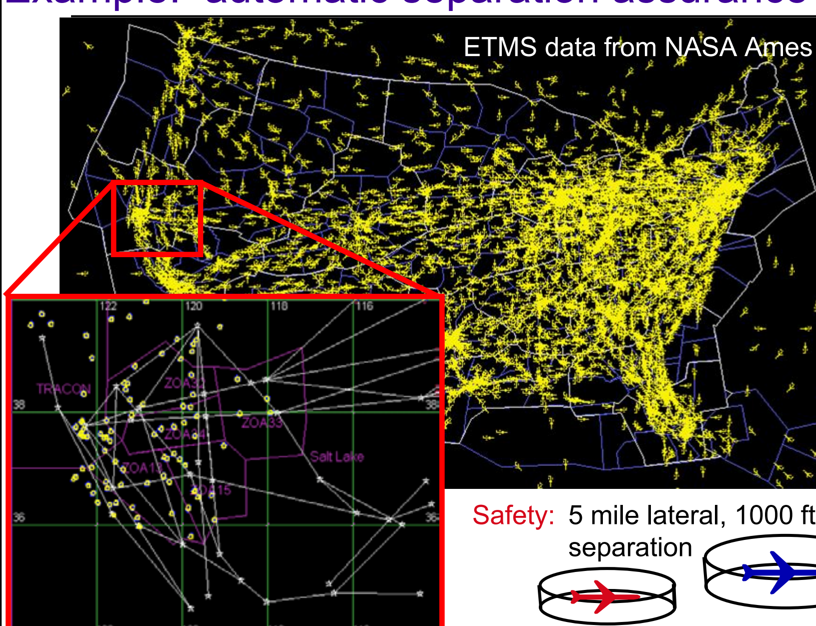
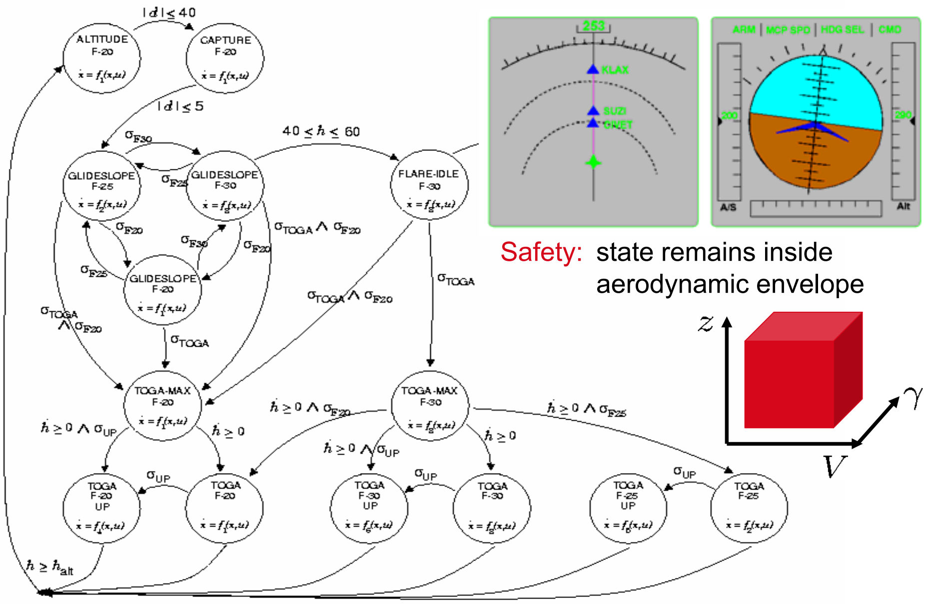
Example: automatic separation assurance in ATC



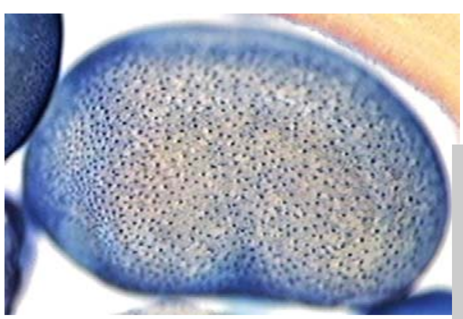
Safety: 5 mile lateral, 1000 ft vertical separation

Example: civil autoland under mode switches

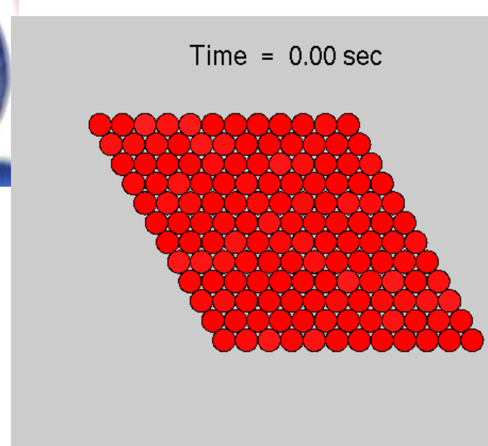


Safety: state remains inside aerodynamic envelope

Example: Cell Differentiation in *Xenopus*



Safety: biologically feasible equilibria are reachable



Hybrid System Model

$H = \{S, Init, In, f, Dom, R\}$

where

$S = Q \cup \Re^n$
state space

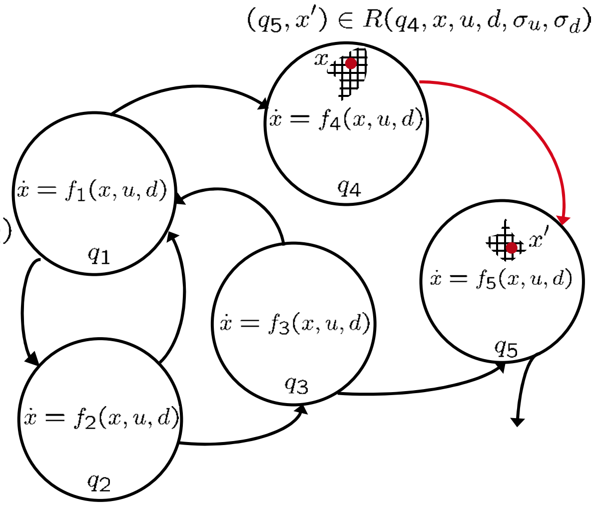
$Init \subseteq S$
initial states

$In = (U \cup D) \cup (\Sigma_u \cup \Sigma_d)$
inputs

$f : S \times In \to S$
vector field

$Dom \subseteq S$
domain

$R : S \times In \to 2^S$
transitions

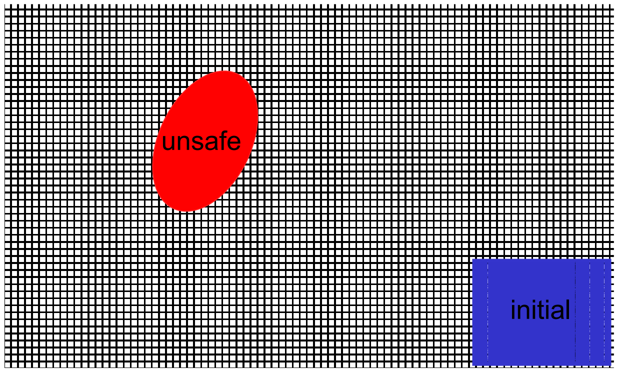


[Lygeros '96, Henzinger '96, Puri & Varaiya '95]

Outline

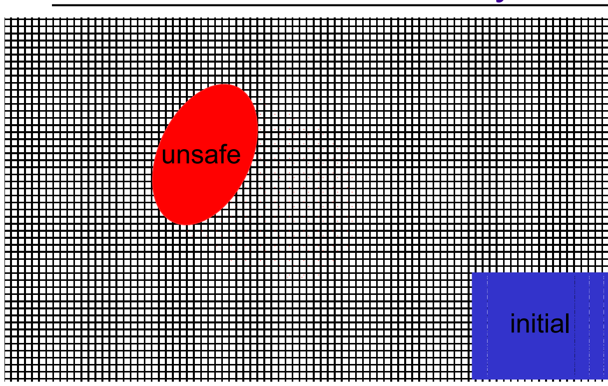
- Controller synthesis for hybrid systems using differential games
  - Algorithm for computing reachable sets and synthesis of controllers
    - for continuous systems
    - for hybrid systems
  - Applications
    - aircraft collision avoidance
    - flight management system design
  - Fast overapproximations
- Controller synthesis using predicate abstraction
  - Analysis of biological regulatory networks

Controller Synthesis



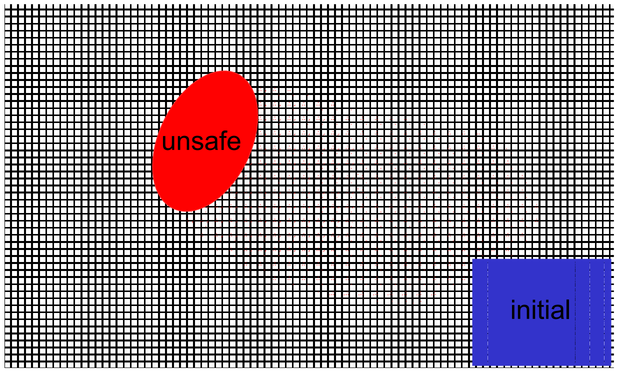
Controller synthesis: to guarantee that a system satisfies a safety property

Controller Synthesis



Controller synthesis: to guarantee that a system satisfies a safety property

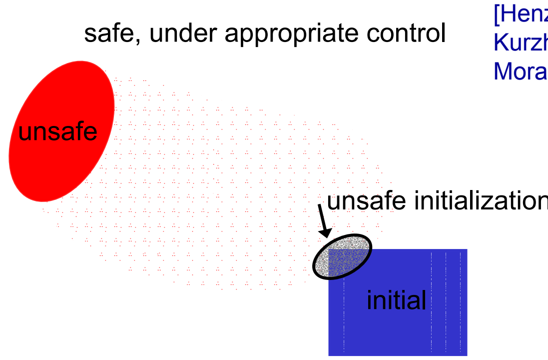
Controller Synthesis



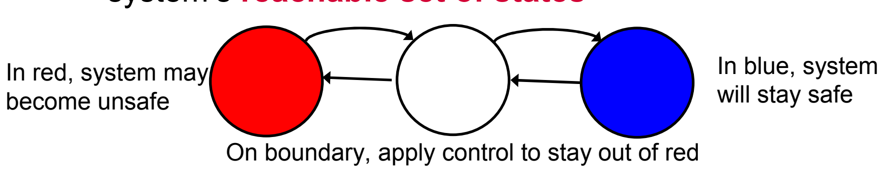
Controller synthesis: to guarantee that a system satisfies a safety property

Controller Synthesis

[Henzinger '96, Krogh '00, Kurzhanski & Varaiya '00, Morari '99]

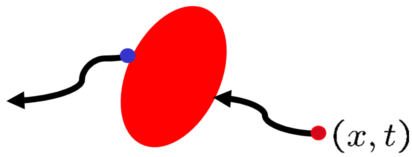


Safety Property can be encoded as a condition on the system's **reachable set of states**



Continuous reach set calculation

$\dot{x} = f(x, u, d)$



$G(0) = \{x : l(x) < 0\}$

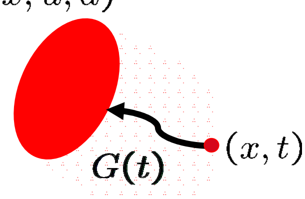
Differential game over $[t, 0]$:

$$V(x, u(\cdot), d(\cdot), t) = l(x(0))$$

$$J(x, t) = \max_u \min_d V(x, u(\cdot), d(\cdot), t)$$

Continuous reach set calculation

$\dot{x} = f(x, u, d)$



$G(t)$ is the set of states for which, for all control actions, there exists a disturbance action which can drive the system to $G(0)$ in at most t

$G(0) = \{x : l(x) < 0\}$

Differential game over $[t, 0]$:

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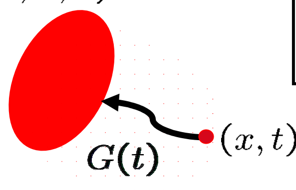


Computation provides, at each (x,t):

$|J(x,t)|$ $\quad \frac{\partial J(x,t)}{\partial x}$

Distance to boundary

Vector pointing to boundary

Theorem [Computing $G(t)$]:

$$G(t) = \{x : J(x,t) < 0\}$$

where $J(x,t)$ is the unique Crandall-Evans-Lions viscosity solution to:

$$-\frac{\partial J(x,t)}{\partial t} = \min\{0, \max_u \min_d \frac{\partial J(x,t)}{\partial x} f(x,u,d)\}$$

[Mitchell, Bayen, Tomlin '02, Tomlin, Lygeros, Sastry '00]

Continuous reach set calculation

Proof [Mitchell, Bayen, Tomlin '02]:

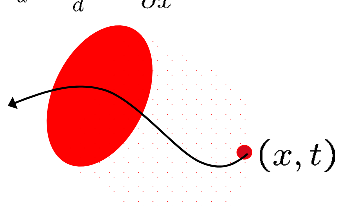
based on results of Evans and Souganidis (1984) for

$$-\frac{\partial J(x,t)}{\partial t} = \max_u \min_{\tilde{d}} \frac{\partial J(x,t)}{\partial x} \tilde{f}(x,u,d)$$

Set $\tilde{d} = \{d, \underline{d}\}$ where $\underline{d} : [t, 0] \to [0, 1]$

and define $\tilde{f}(x,u,\tilde{d}) = \underline{d} f(x,u,d)$

$$\begin{aligned}\max_u \min_{\tilde{d}} \frac{\partial J(x,t)}{\partial x} \tilde{f}(x,u,\tilde{d}) &= \max_u \min_d \min_{\underline{d} \in [0,1]} \frac{\partial J(x,t)}{\partial x} \underline{d} f(x,u,d) \\ &= \min\{0, \max_u \min_d \frac{\partial J(x,t)}{\partial x} f(x,u,d)\}\end{aligned}$$



Continuous reach set calculation

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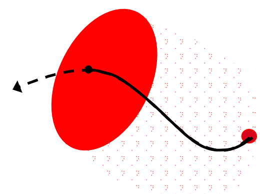
$$= \min\{0, \max_u \min_d \frac{\partial J(x,t)}{\partial x} f(x,u,d)\}$$



Numerical computation of reach sets

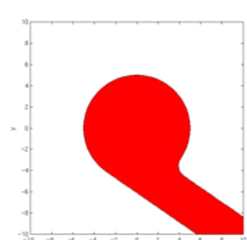
Create a level set function $J(x,t)$ such that:

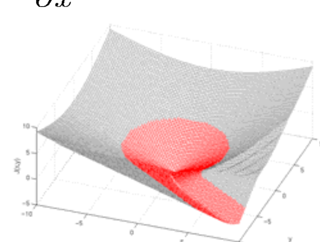
- Boundary of region is defined implicitly by $J(x,t) = 0$
- $|J(x,t)|$ is the distance from x to the boundary at time t
- J is negative inside region and positive outside

Propagating regions with level sets:

- In our problem, the evolution of $J(x,t)$ is governed by:

$$-\frac{\partial J(x,t)}{\partial t} = \max_u \min_d \frac{\partial J(x,t)}{\partial x} F(x,u,d)$$





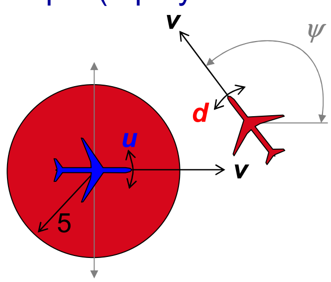
[based on Osher and Sethian, 1988]

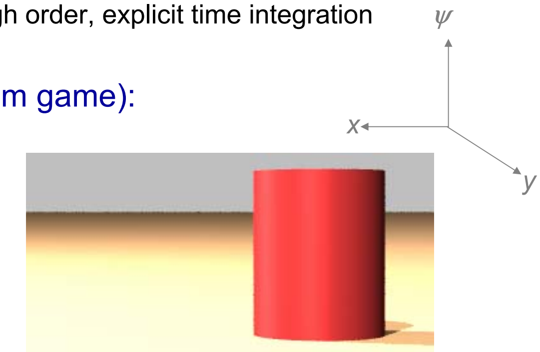
Numerical computation of reach sets

Level set methods:

- Convergent numerical algorithms to compute viscosity solution
- Non-oscillatory, high accuracy spatial derivative approximation
- Stable, consistent numerical Hamiltonian
- Variation diminishing, high order, explicit time integration

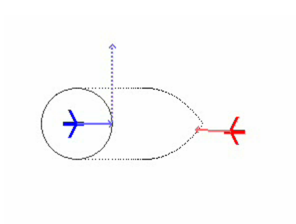
Example (2 player zero sum game):

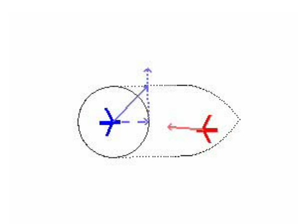




[Mitchell, Tomlin '01]

Collision Avoidance Control

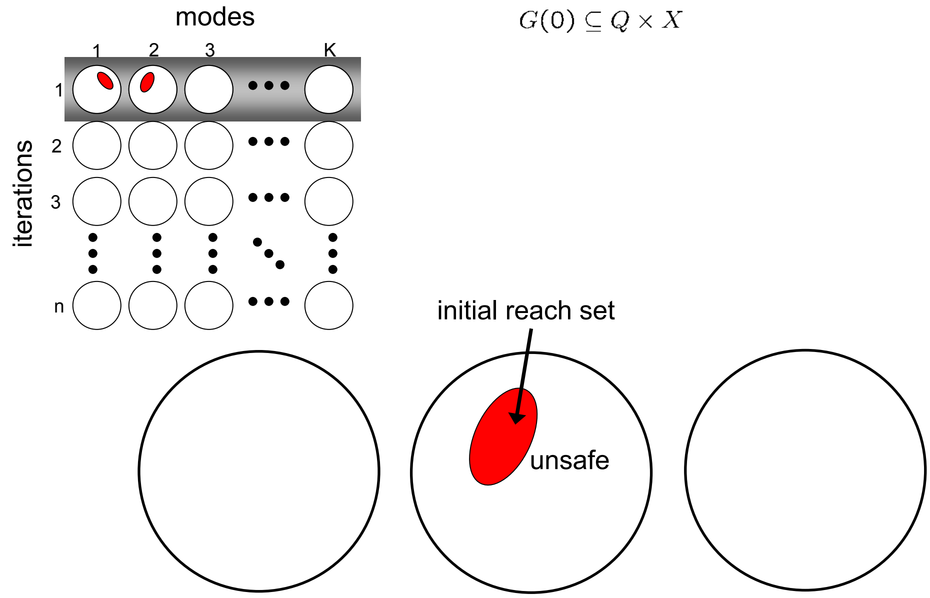




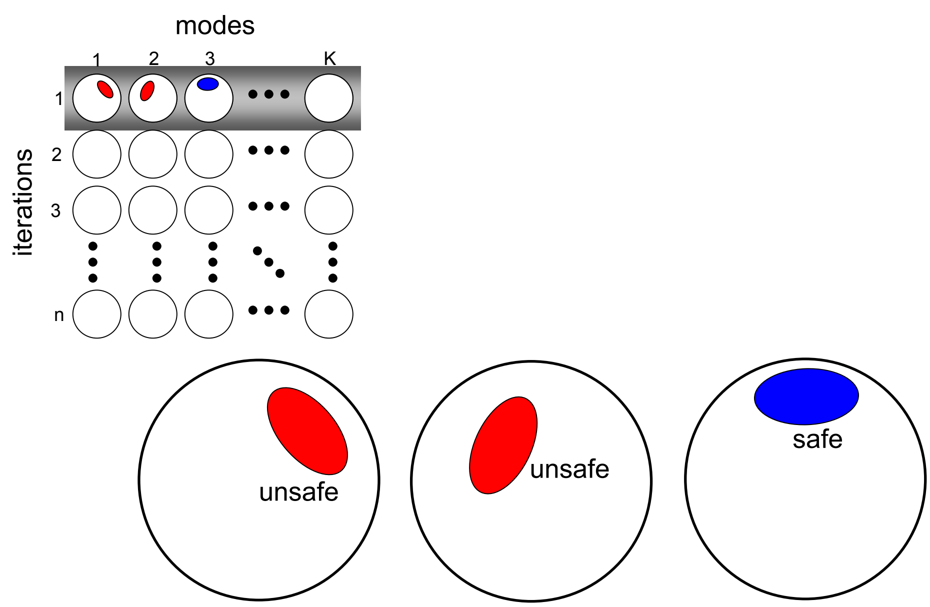
[Mitchell, Tomlin '01]

Computing Reach Sets for Hybrid Systems

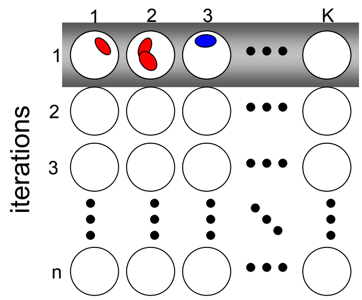
$G(0) \subseteq Q \times X$



Reach Sets: Initialize

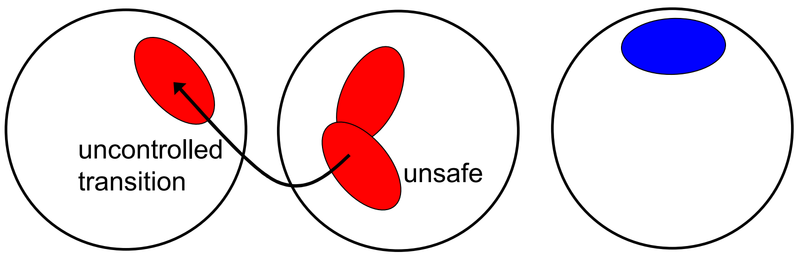


Reach Sets: uncontrollable predecessor

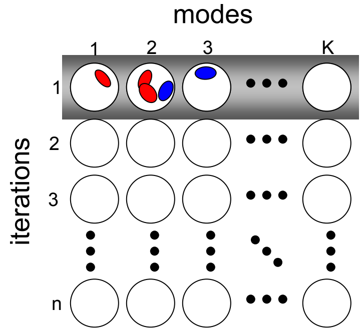


$$Pre_d(K^c) \;=\; \{(q,x) : \forall(u,\sigma_u)\;\exists(d,\sigma_d)\; R(q,x,u,d,\sigma_u,\sigma_d) \cap K^c \neq 0\} \cup K^c$$

$K \subseteq Q \times X$ "safe"

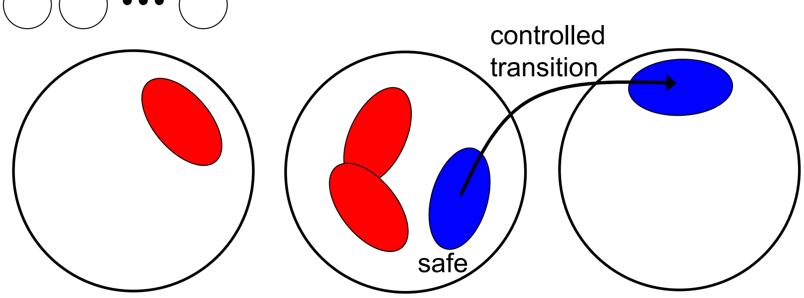


Reach Sets: controllable predecessor

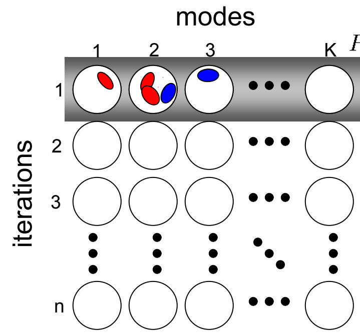


$$Pre_u(K) \;=\; \{(q,x) : \exists(u,\sigma_u)\;\forall(d,\sigma_d)\; R(q,x,u,d,\sigma_u,\sigma_d) \subseteq K\}$$

$K \subseteq Q \times X$ "safe"



Reach Sets: Variational Inequality



States which reach G without hitting E first:

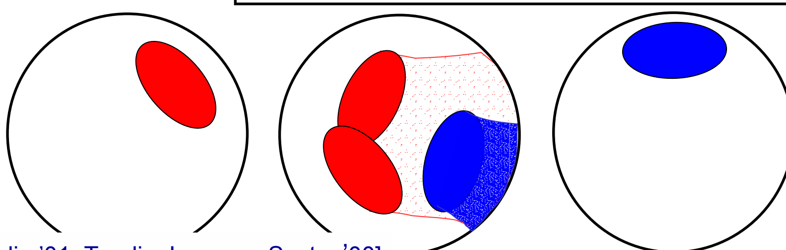
$$Reach(G,E) = \{(q,x) : \forall u \exists d \; \exists t \geq 0 \; (q,x(t)) \in G$$
$$(q,x(s)) \in Dom \backslash E \;\; \forall s \in [0,t]\}$$

$$Reach(G,E) = \{x : J_G(x,t) \leq 0\}$$
$$E = \{x : J_E(x) \leq 0\}$$

where

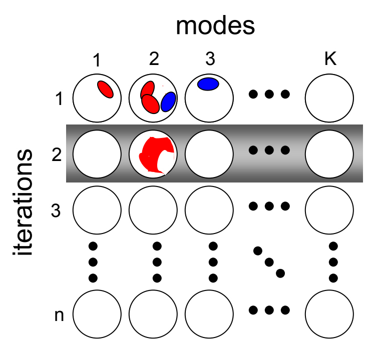
$$\frac{\partial J_G(x,t)}{\partial t} + \min\{0, \max_u \min_d \frac{\partial J_G(x,t)}{\partial x} f(x,u,d)\} = 0$$

subject to $J_G(x,t) \geq -J_E(x)$



[Mitchell, Tomlin '01, Tomlin, Lygeros, Sastry '00]

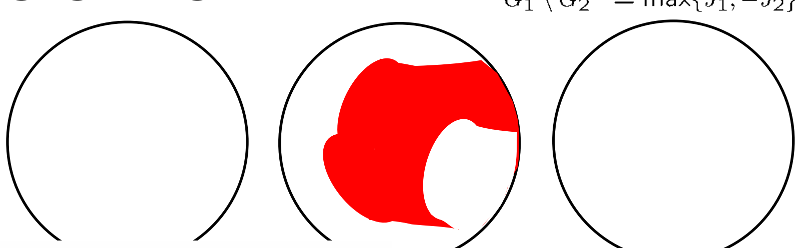
Reach Sets: Iterate



Initialize: $W^0 = G(0)^c, W^1 = 0, i = 0$
While $W^i \neq W^{i+1}$ do
$W^{i-1} = W^i \setminus Reach(Pre_d((W^i)^c), Pre_u(W^i))$
$i = i - 1$
end while

$$G_1(t) = \{x : J_1(x,t) < 0\}$$
$$G_2(t) = \{x : J_2(x,t) < 0\}$$

$$G_1 \cup G_2 = \min\{J_1, J_2\}$$
$$G_1 \cap G_2 = \max\{J_1, J_2\}$$
$$G_1 \setminus G_2 = \max\{J_1, -J_2\}$$



[Mitchell, Tomlin '01, Tomlin, Lygeros, Sastry '00]

Outline

- Controller synthesis for hybrid systems using differential games
  - Algorithm for computing reachable sets and synthesis of controllers
    - for continuous systems
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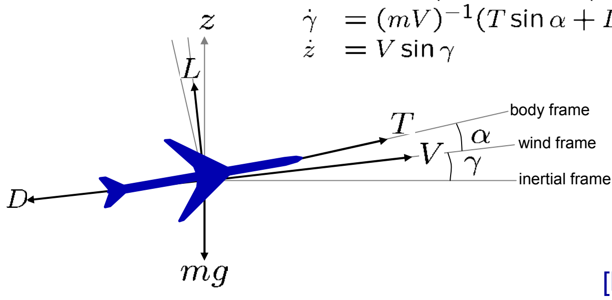
  - Applications
    - flight management system design
    - aircraft collision avoidance
  - Fast overapproximations
- Controller synthesis using predicate abstraction
  - Analysis of biological regulatory networks

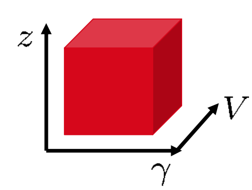
Example: Aircraft Autolander

Aircraft must stay within safe flight envelope during landing:

- Bounds on velocity (V), flight path angle (γ), height (z)
- Control over engine thrust (T), angle of attack (α), flap settings
- Model flap settings as discrete modes of hybrid automata
- Terms in continuous dynamics may depend on flap setting

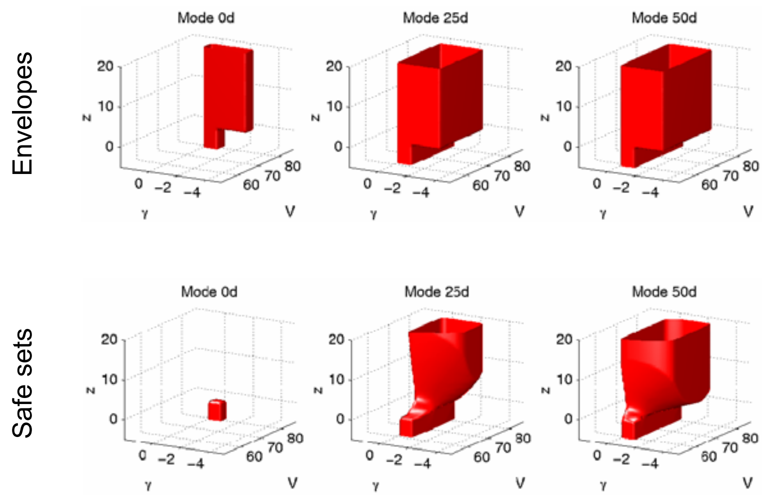
$$\begin{aligned} \dot{V} &= m^{-1}(T\cos\alpha - D(\alpha, V) - mg\sin\gamma) \\ \dot{\gamma} &= (mV)^{-1}(T\sin\alpha + L(\alpha, V) - mg\cos\gamma) \\ \dot{z} &= V\sin\gamma \end{aligned}$$



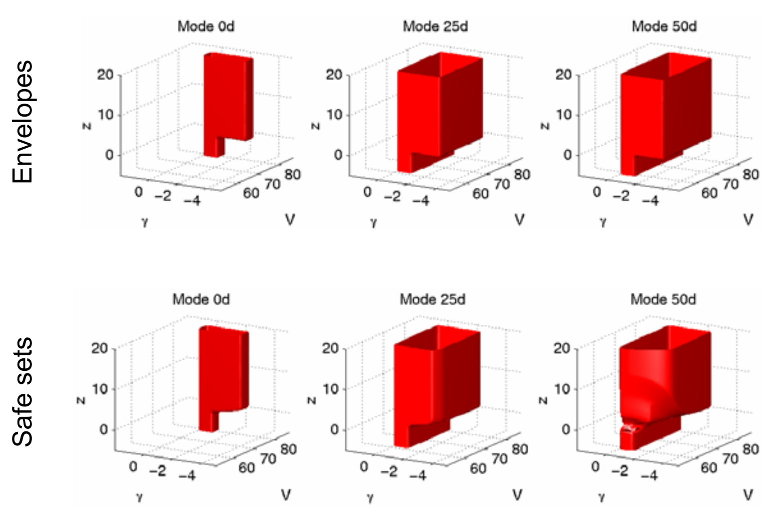


[Mitchell, Bayen, Tomlin '01]

Landing Example: No Mode Switches



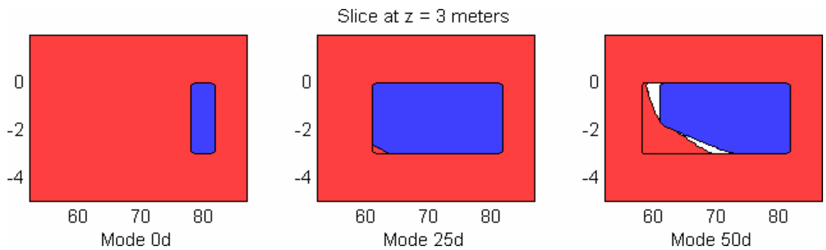
Landing Example: Mode Switches



Landing Example: Synthesizing Control

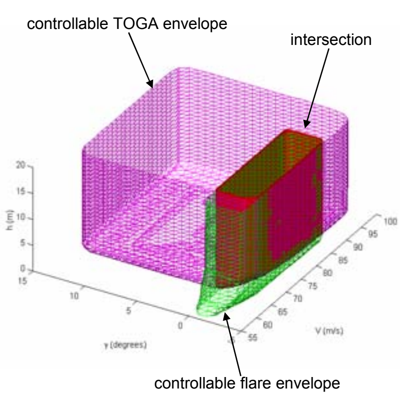
For states at the boundary of the safe set, results of reach-avoid computation determine

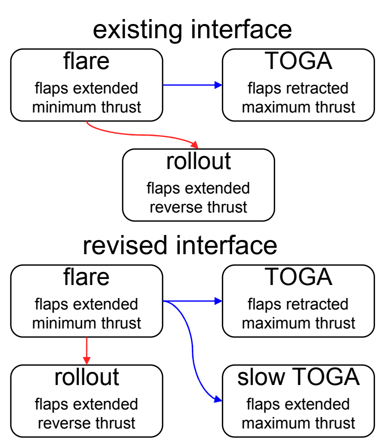
- What continuous inputs (if any) maintain safety
- What discrete jumps (if any) are safe to perform
- Level set values and gradients provide all relevant data



Application to Autoland Interface

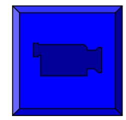
- Controllable flight envelopes for landing and Take Off / Go Around (TOGA) maneuvers may not be the same
- Pilot's cockpit display may not contain sufficient information to distinguish whether TOGA can be initiated



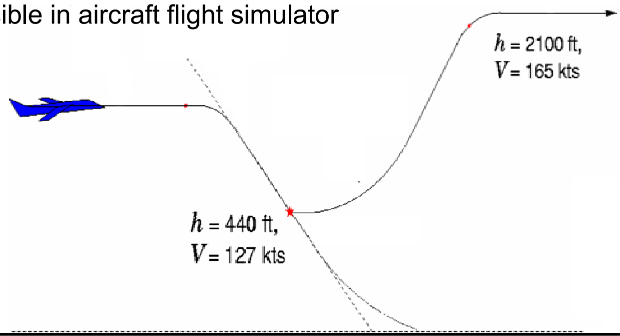


Aircraft Simulator Tests

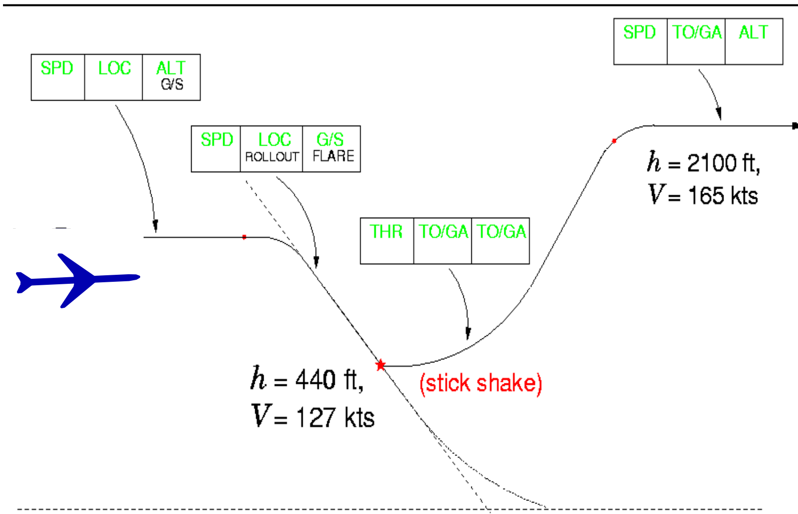
- Setup
  - Commercial flight simulator, B767 pilot
  - Digital video of primary flight display
- Maneuver
  - Go-around at low speed, high descent rate
- Goal
  - Determine whether problematic behavior predicted by our model is possible in aircraft flight simulator



(movie)



Aircraft Simulator Results

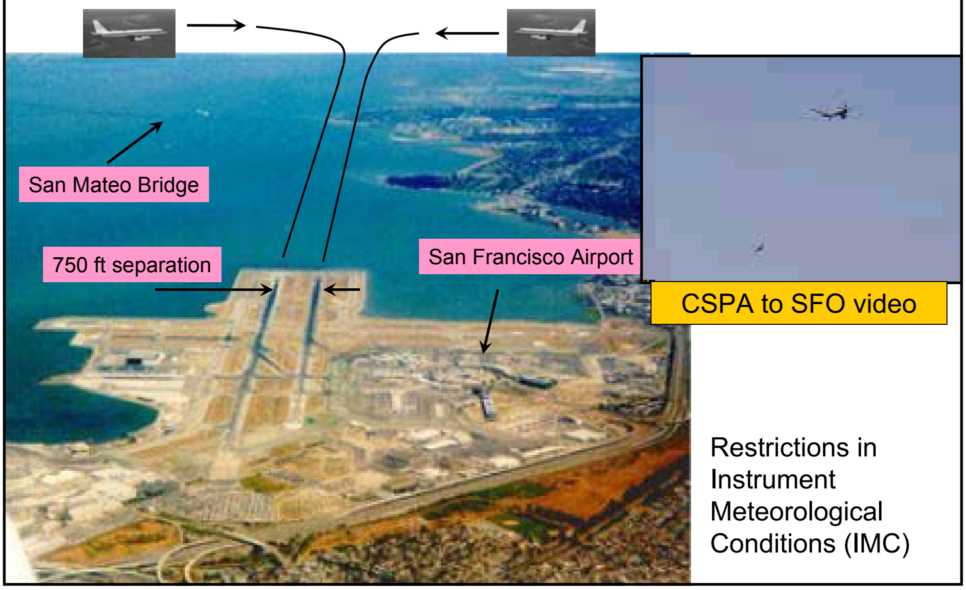


Produced unexpected behavior

Non-standard procedure; Unable to duplicate

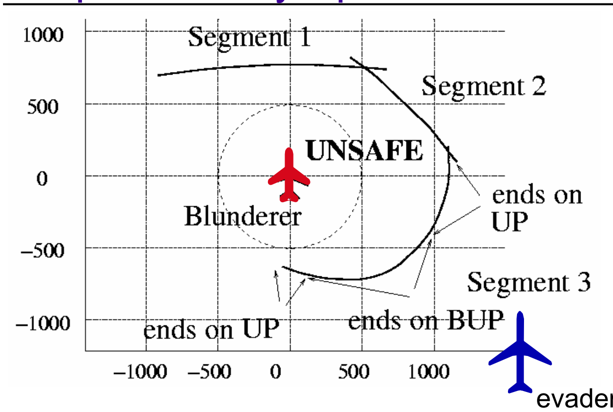
Validated types of problems addressed by this method

Example: Closely Spaced Parallel Approaches



Restrictions in Instrument Meteorological Conditions (IMC)

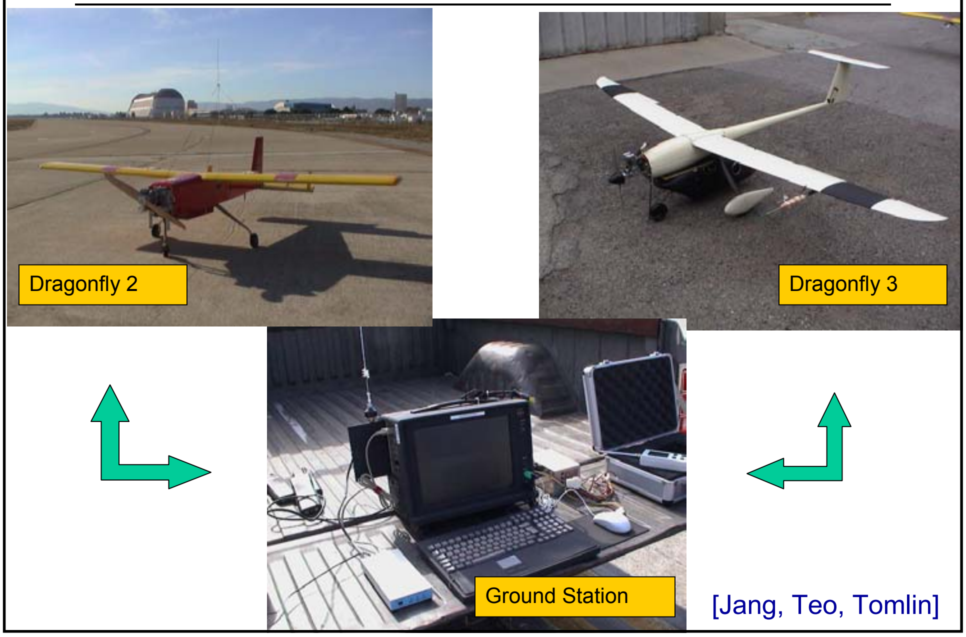
Example: Closely Spaced Parallel Approaches



Three emergency escape maneuvers (EEMs):

1. Evader accelerates straight ahead
2. Evader accelerates, turns to the right 45 deg
3. Evader turns to the right 60 deg

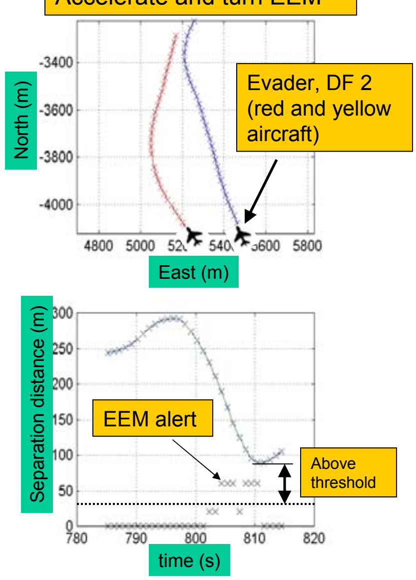
Tested on the Stanford DragonFly UAVs



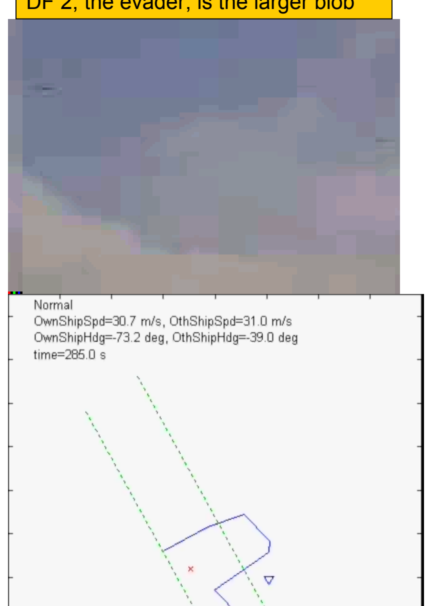
[Jang, Teo, Tomlin]

Flight Demo 1 -- Sept 2003

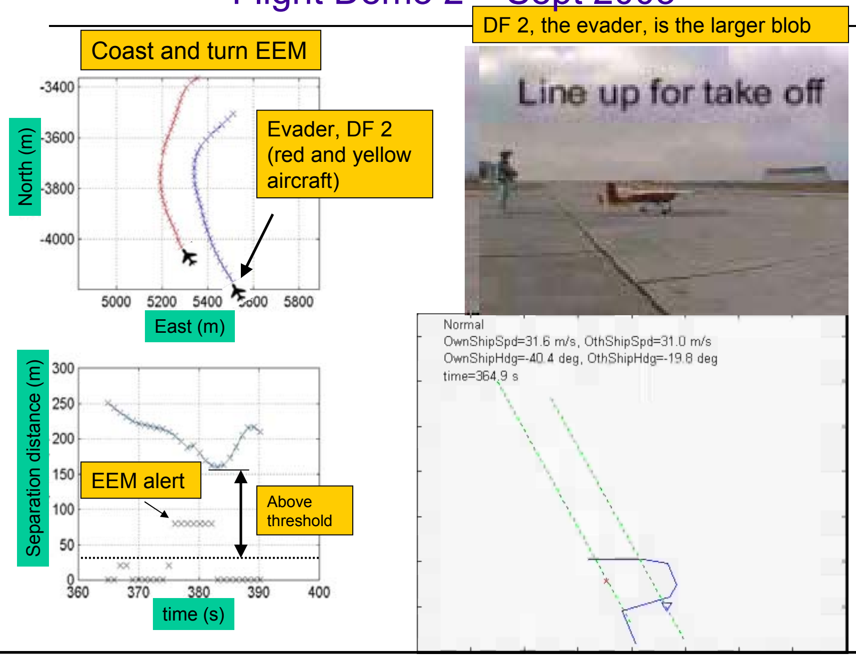
Accelerate and turn EEM



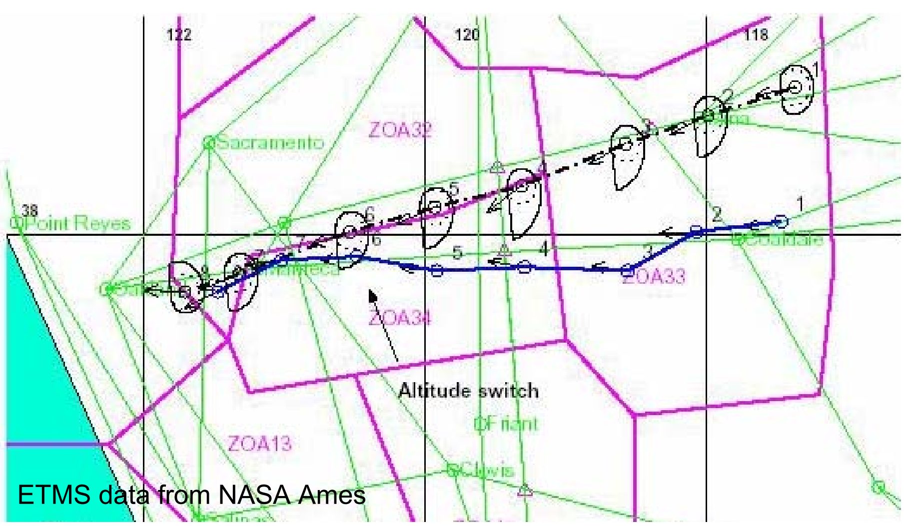
DF 2, the evader, is the larger blob



Flight Demo 2 – Sept 2003



Collision Avoidance Control



[Mitchell, Bayen, Tomlin '03]

Overapproximating Reachable Sets

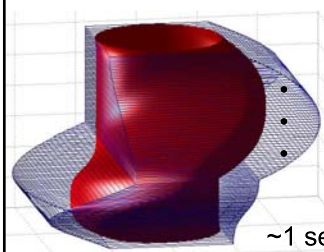
Exact: $$\frac{\partial J(x,t)}{\partial t} + \min\{0, \max_u \min_d \frac{\partial J(x,t)}{\partial x} f(x,u,d)\} = 0$$

Approximate: $$\frac{\partial J^+(x,t)}{\partial t} + \min\{0, \max_u \min_d \frac{\partial J^+(x,t)}{\partial x} f(x,u,d)\} \leq \mu(t)$$

Overapproximative reachable set:

$$V^+(\tau) \equiv \{x | J^+(x,\tau) \leq \int_0^\tau \mu(t)dt + \max_{x(0)} J^+(x(0),0)\}$$

[Khrustalev, Varaiya, Kurzhanski]



- Polytopic overapproximations for nonlinear games
- Subsystem level set functions
- “norm-like” functions with identical strategies to exact

[Hwang, Stipanovic, Tomlin]

~1 sec on 700MHz Pentium III (vs 4 minutes for exact)

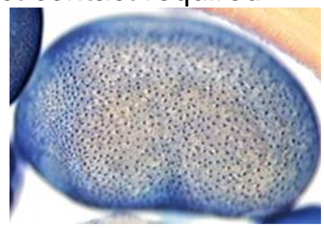
Outline

- Controller synthesis for hybrid systems using differential games
  - Algorithm for computing reachable sets and synthesis of controllers
    - for continuous systems
    - for hybrid systems
  - Applications
    - aircraft collision avoidance
    - flight management system design
  - Fast overapproximations
- Controller synthesis using predicate abstraction
  - Analysis of biological regulatory networks

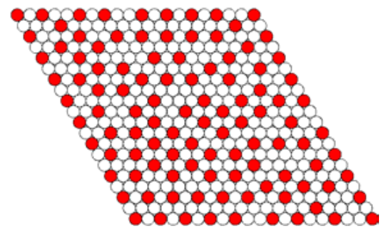
Delta-Notch Signaling Pathway

Observables:

- Transmembrane proteins: direct contact required
- **Notch**
  - Receptor $\rightarrow$ production promoted by localized high Delta concentrations



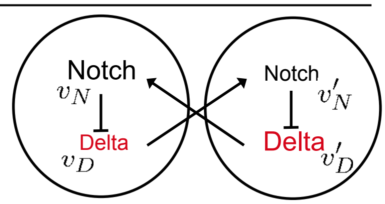
- **Delta**
  - Ligand $\rightarrow$ activates Notch, production promoted by low intracellular Notch concentrations
- Produces lateral inhibition
- **Delta** ⟶ hair



[“Deconstructing Hairy”, Ghosh, Tomlin ‘01]

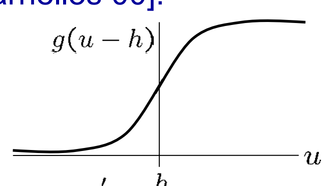
Previous work

- Accepted “influence” model:



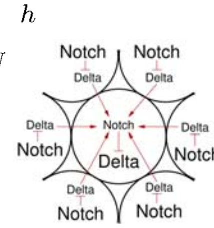
- Reaction-diffusion [Collier 96, Marnellos 00]:

$$g(u-h) = 0.5\left(1 + \frac{u-h}{\sqrt{1+(u-h)^2}}\right)$$

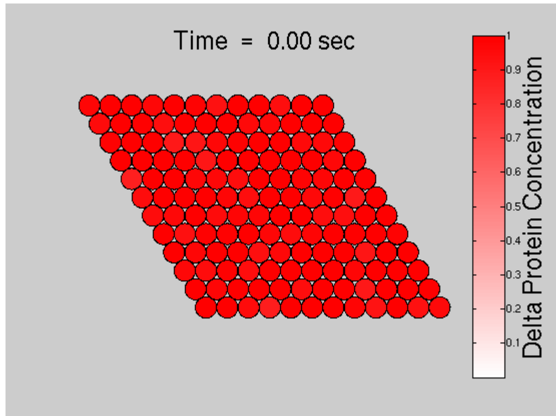


$$\dot{v}_N = \lambda_N v_N + g(u_N - h_N)R_N \quad \text{where } u_N = v'_D$$
$$\dot{v}_D = \lambda_D v_D + g(u_D - h_D)R_D \quad \text{where } u_D = -v_N$$

- Parameters λ_i, h_i, R_i are unknown

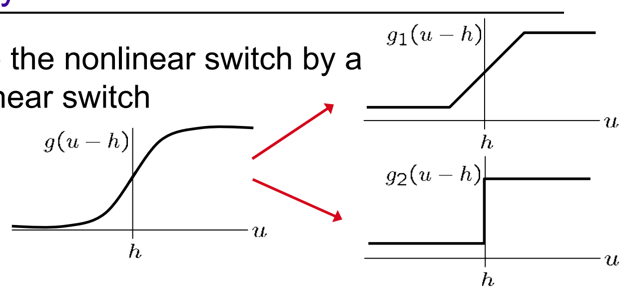


Parameters may be hard to identify

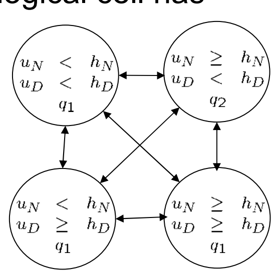


Hybrid Model of a Cell

Idea: approximate the nonlinear switch by a piecewise linear switch



- For example, using g_2, each biological cell has four discrete states:
  - o Notch off/delta off
  - o Notch on/delta off
  - o Notch off/delta on
  - o Notch on/delta on



$$\dot{v}_N = \lambda_N v_N + g(u_N - h_N) R_N$$
$$\dot{v}_D = \lambda_D v_D + g(u_D - h_D) R_D$$

[Ghosh, Tomlin '01]

Compute reach sets for the (large) hybrid model

Idea:

- compute reach sets symbolically, in terms of parameters λ_i, h_i, R_i from the biologically feasible equilibrium patterns
- determine constraints on these parameters and initial conditions so that these biologically feasible equilibria are reachable

Problem: large state space

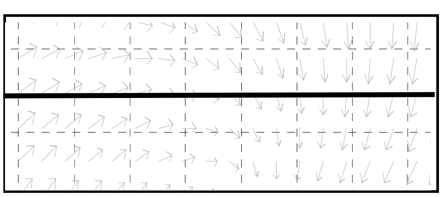
- 1 cell: 2 continuous states, 4 discrete states
- 2 cell: 4 continuous states, 16 discrete states
- 4 cell: 8 continuous states, 256 discrete states
- 9 cell: 18 continuous states…

Reach set calculation for piecewise affine systems

$A_i, b_i, \alpha_i, \beta_i$ are symbolic

A_i diagonal

A simple example:

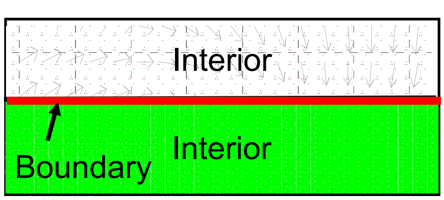


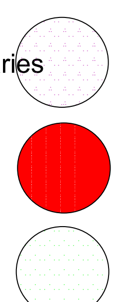
$\dot{x} = A_1 x + b_1$

$x_1 + \alpha_1 x_2 + \beta_1 = 0$

$\dot{x} = A_2 x + b_2$

Step 1: Separate partitions into interiors and boundaries



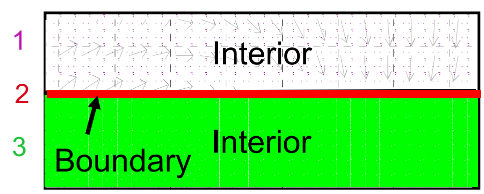


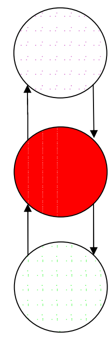
[Ghosh, Tomlin '04]

Reach set calculation for piecewise affine systems

Step 2: Compute transitions between modes. For example, in mode 1:

- Determine direction of flow across the boundary
- Compute sign of Lie derivative of function describing boundary, with respect to mode 1 dynamics: $\mathcal{L}_{A_1 x + b_1}(x_1 + \alpha_1 x_2 + \beta_1)$
- If $\mathcal{L} < 0$ then flow is from mode 1 to mode 2
- If $\mathcal{L} > 0$ then flow is from mode 2 to mode 1
- If $\mathcal{L} = 0$ then flow remains on boundary



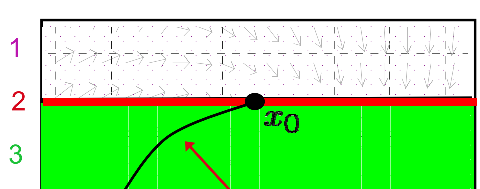


[Ghosh, Tomlin ‘04]

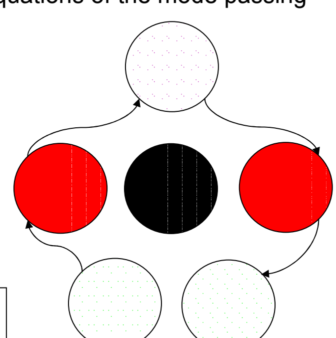
Reach set calculation for piecewise affine systems

Step 3: Subpartition modes that have more than one exit transition.

- In Mode 2, split the mode at the point of intersection or inflexion (where $\mathcal{L} = 0$)
- In Mode 3, partition between those states which remain in 3 and those which enter mode 2. The separation line (or surface) is the analytical solution of the differential equations of the mode passing through the separation point.



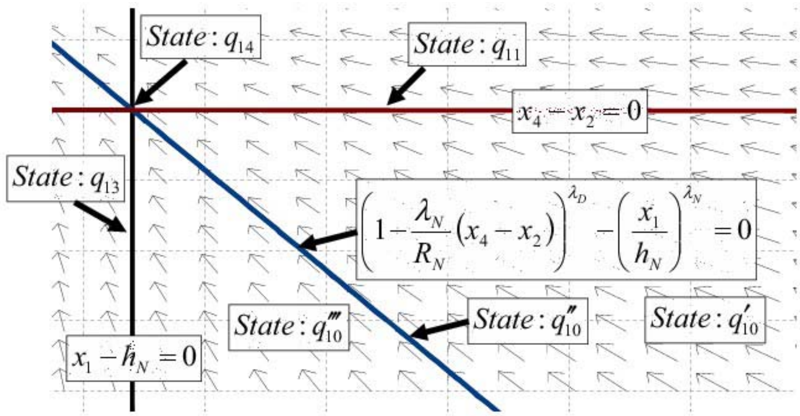
$$x = exp(A_2 t)x_0 + \int_0^t exp(A_2 \tau) b_2 d\tau$$



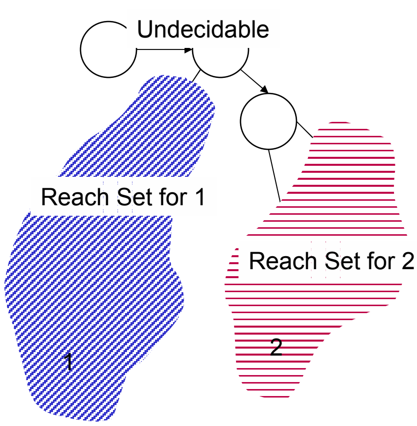
time t is eliminated to form a closed form polynomial expression → $x_1^a + x_2^b = c$, $x_1 - x_3 = 0$

[Ghosh, Tomlin ‘04]

Reach set calculation for piecewise affine systems

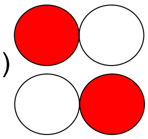


Resulting discrete transition system



Even if a bisimulation cannot be computed, the algorithm constructs an abstraction from which an underapproximation of the backwards reachable set from the equilibria can be determined.

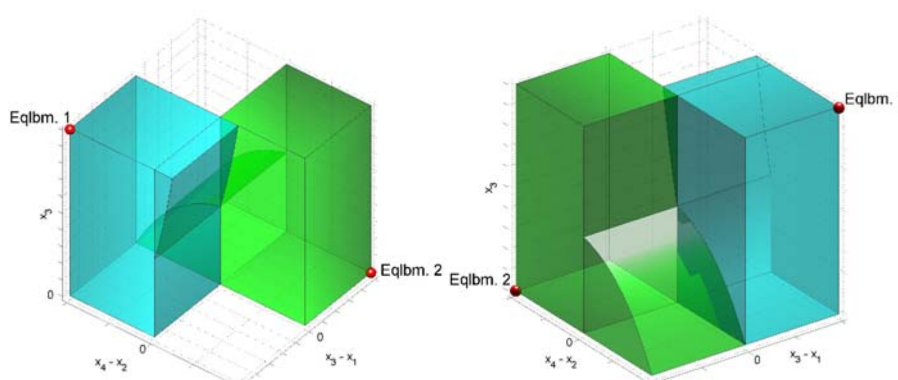
Reach Set Results: Two Cell Automaton



- 4 continuous variables: `x1,x3` (Delta); `x2,x4` (Notch)
- 16 discrete states
- 2 biologically observed equilibria:
  - One cell has high Delta protein and low Notch and the other cell has low Delta and high Notch protein, and the converse
- Backward reachable set computed from both equilibria

- Observations:
  - Conditions on variables: $h_D, h_N : -\frac{R_N}{\lambda_N} < h_D \leq 0 \wedge 0 < h_N \leq \frac{R_D}{\lambda_D}$
  - All initial conditions that satisfy `x3>x1 AND x4<x2` converge to one equilibrium and all that satisfy `x3<x1 AND x4>x2` converge to the other
  - The separatrix `x3 = x1` separating the reach sets is clearly visible in the two dimensional projection of the reach sets on the `x1, x3` axes

Visualization of Reach Sets for 2-cell

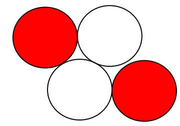


Implementation:

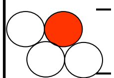
- Symbolic/string manipulations in MATLAB
- Decision procedure on polynomials done in QEPCAD

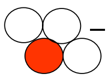
[Ghosh, Tomlin '04]

Reach Set: Four Cell Automaton



- 8 continuous variables: `x1,x3,x5,x7` (Delta)
  `x2,x4,x6,x8` (Notch)
- 256 discrete states
- 3 possible equilibria:



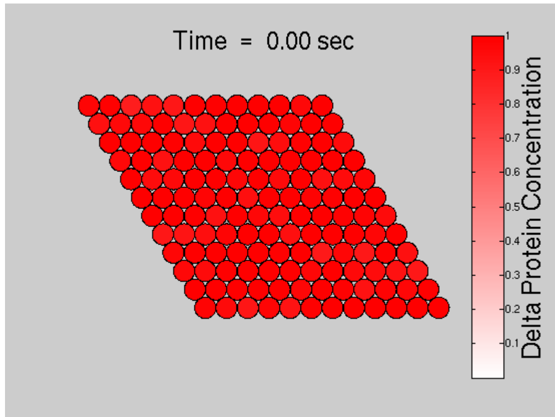


  - cells 1 and 4 have high Delta levels and low Notch levels, and cells 2 and 3 have high Notch levels and low Delta levels
  - cell 2 has high Delta levels and low Notch levels at steady state, and all the other cells have low Delta levels and high Notch levels
  - cell 3 has high Delta concentration and low Notch concentration at steady state and the other three cells have low Delta levels and high Notch levels
- Model-checker: 8 hours CPU time on Pentium III 500 MHz
- Resultant reach set is complex and difficult to visualize

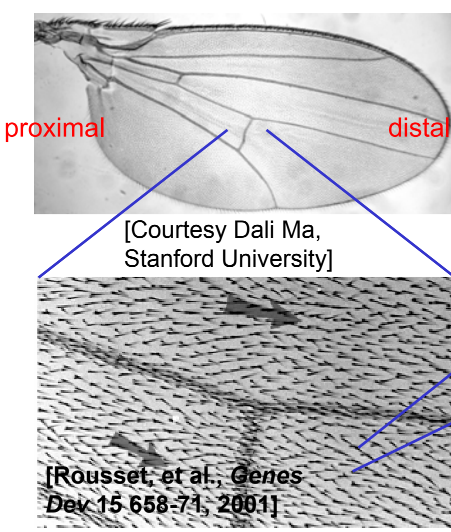
Reach Set: Four Cell Automaton

- Interesting observations:
  - Among the hybrid states that are completely reachable, fourteen states satisfy the constraint `x7+x3+x1-hN<0 AND x7+x5+x1-hN<0 AND x5+x3-hN<0`. This implies that for all Notch protein levels except `-hD>=x2 AND -hD>=x4 AND -hD>=x6`, if the Delta levels satisfy the constraint, then the equilibrium will be attained.
  - It is observed that all hybrid states with `-hD<x2 AND x5+x3<hN AND x7+x3+x1>=hN` are completely unreachable. This implies that regardless of the Notch protein levels of cells 2, 3 and 4, if the Notch protein concentration of cell 1 and the Delta protein levels of all four cells satisfy the above constraints, then the equilibrium will be unreachable.

Simulation using viable parameters and IC

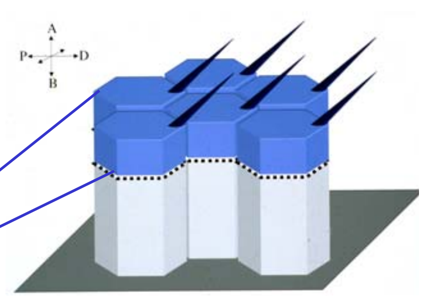


Drosophila wing epithelium



[Courtesy Dali Ma, Stanford University]

- *Drosophila* wing hairs point distally, virtually error free
- Planar Cell Polarity (PCP)

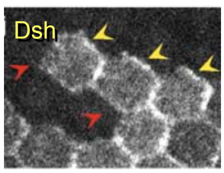


Human pathology:
cochlear hair cells
spina bifida
oncogenic Wnt pathway

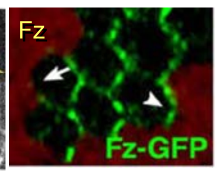
Amonlirdviman, Axelrod, Tomlin '04

Signaling Molecules

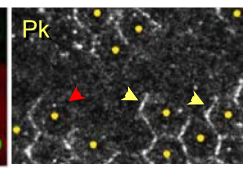
- Includes *Frizzled* (Fz), *Dishevelled* (Dsh), *Prickle* (Pk), *Flamingo* (Fmi) and *Van Gogh* (Vang)
- Dsh and Fz localize on the distal portion of each cell
- Pk and Vang localize on the proximal portion of each cell
- Hair grows at Dsh localization



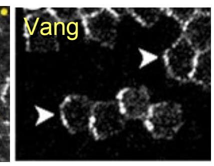
[Axelrod, *Genes Dev* **15**, 1182-7, 2001]



[Strutt, *Molecular Cell* **7**, 367-75, 2001]



[Tree, et al., *Cell* **109**, 371-81, 2002]



[Bastock, et. al., *Development* **130**, 3007-3014, 2003]

Mutant Wings

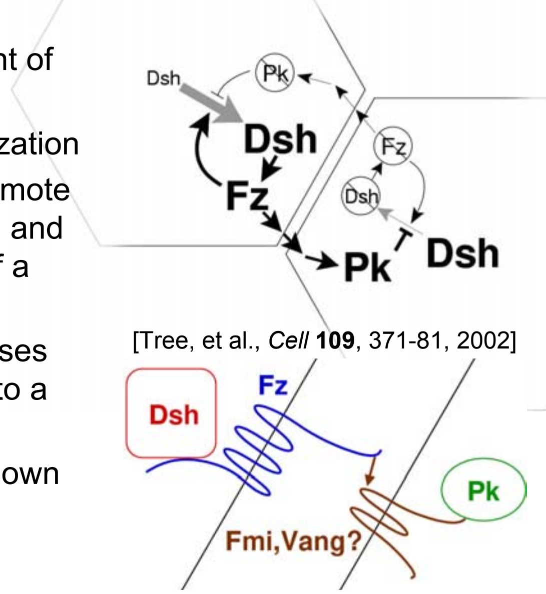
- Loss of Fz disrupts polarity in distal non-mutant cells
- Loss of Vang disrupts polarity in proximal non-mutant cells

[Vinson and Adler, *Nature* **329**, 549-51, 1987]

[Taylor, et al., *Genetics* **150**, 199-210, 1998]

Influence model

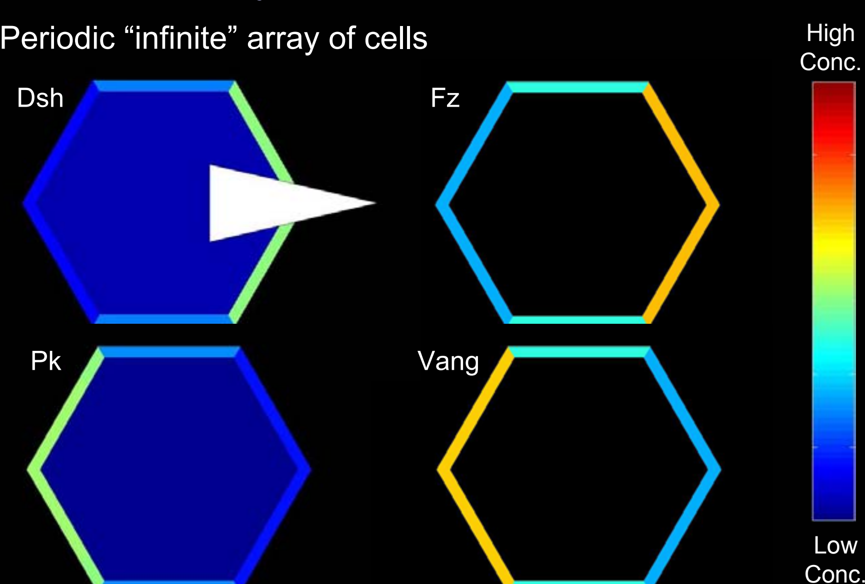
- Fz promotes recruitment of Dsh to a membrane
- Dsh stabilizes Fz localization
- Fz acts indirectly to promote the localization of Vang and Pk on the membrane of a neighboring cell
- Pk (and Vang) suppresses the recruitment of Dsh to a membrane
- Network amplifies unknown directional cue



[Tree, et al., *Cell* **109**, 371-81, 2002]

Wild-type Numerical Results

Periodic “infinite” array of cells



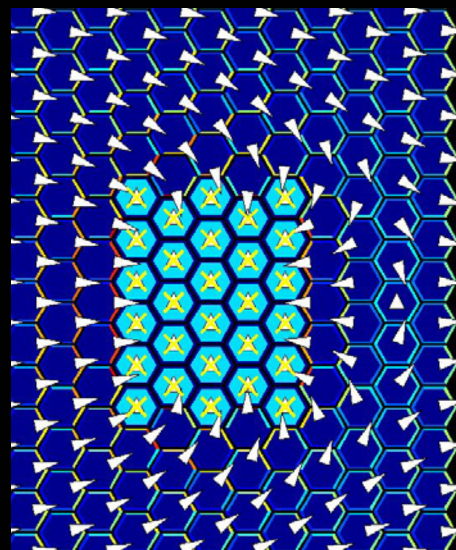
[Amonlirdviman, Axelrod, Tomlin ‘04]

Loss-of-Fz Numerical Results

Dsh Distribution w/ Resulting Hair Pattern

Domineering non-autonomy distal of cloned mutant cells

[Vinson and Adler, *Nature* **329**, 549-51, 1987]



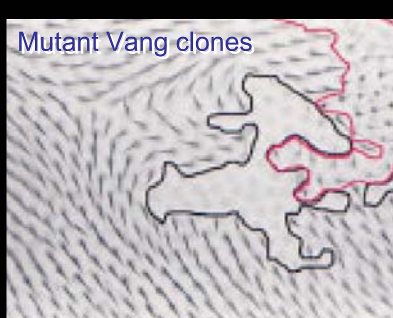
X Fz mutant clone

[Amonlirdviman, Axelrod, Tomlin '04]

Loss-of-Pk (Vang) Numerical Results

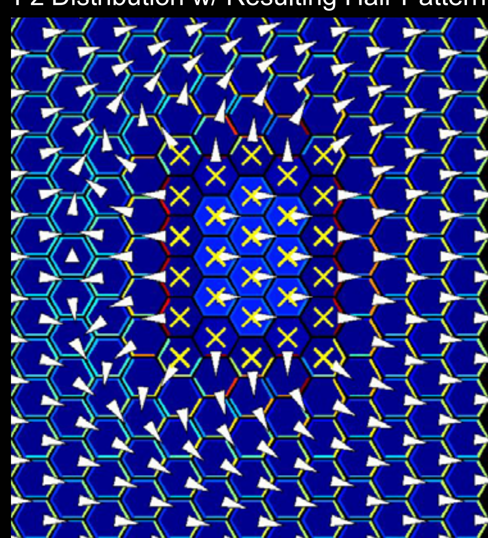
Fz Distribution w/ Resulting Hair Pattern

Domineering non-autonomy proximal of cloned mutant cells



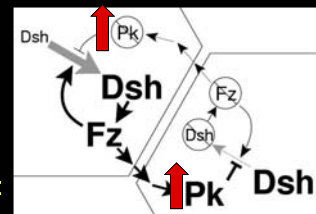
[Taylor, et al., *Genetics* **150**, 199-210, 1998]

X Vang mutant clone



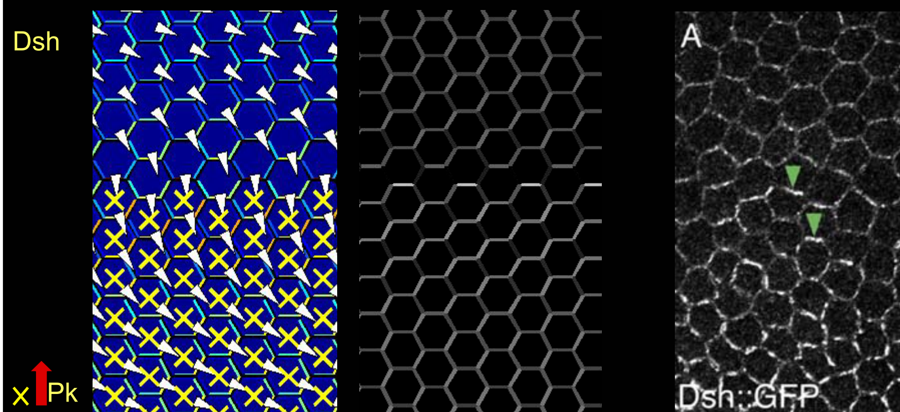
[Amonlirdviman, Axelrod, Tomlin '04]

Biological Insights

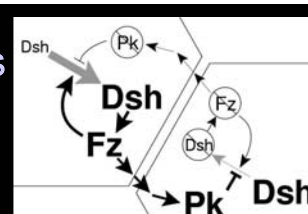


- Demonstrates sufficiency
- Explains even non-intuitive results

Suppose you overexpress Pk in part of the wing:

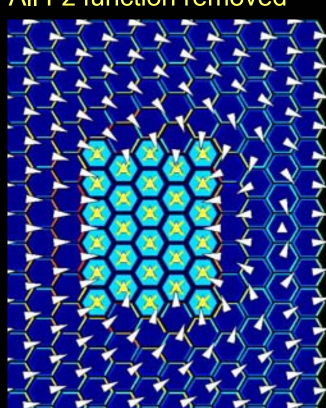


Biological Insights

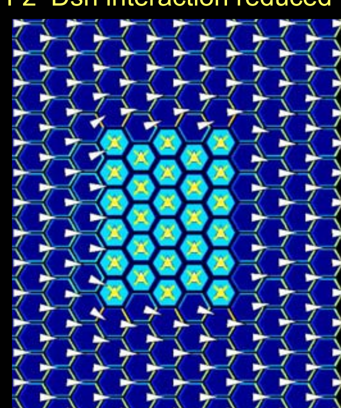


- Suggests mechanism for explaining phenotypes of different mutant Fz alleles – experimentally verifiable

Fz non-autonomous allele –
All Fz function removed

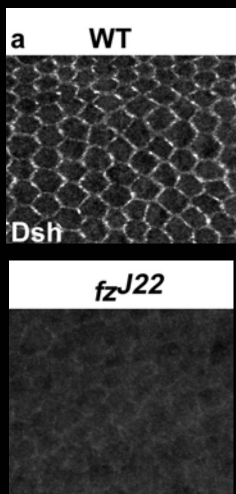


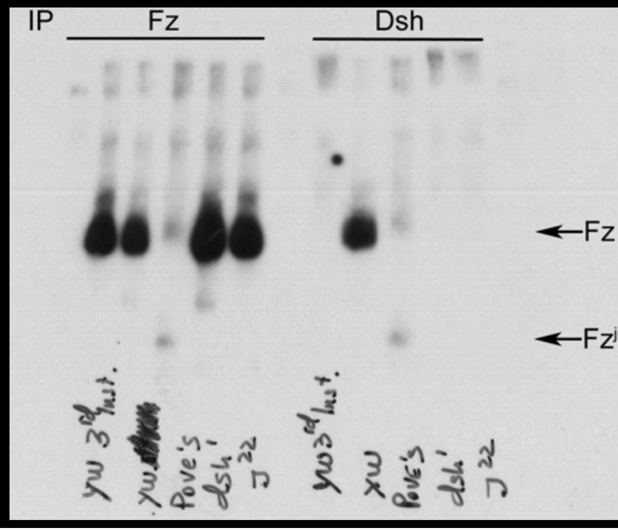
Fz autonomous allele –
Fz–Dsh interaction reduced to 0%



Biological Insights

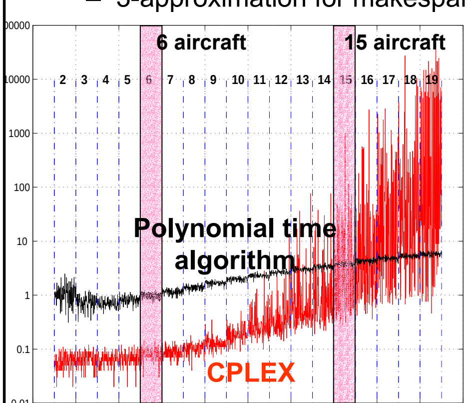
- Suggests mechanism for explaining phenotypes of different mutant Fz alleles – experimentally verifiable

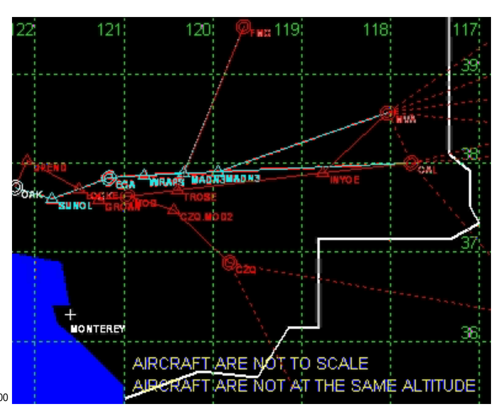




Other research: Approximation algorithms for hybrid systems

- Approximation algorithms for hybrid trajectory optimization
- Applied to routing/scheduling aircraft in vicinities of airports
- Results [Bayen, Zhang, Ye, Tomlin '03,'04]:
  - 5-approximation for minimum sum of arrival times
  - 3-approximation for makespan





Other research: Decentralized optimization

Decentralized optimization using dual decomposition:

primal

Minimize $\sum_{i=1}^{N} f_i(x_i) + h(x_1, x_2, \ldots, x_N)$

Subject to $x_i \in \mathcal{D}_i$, $i = 1, \ldots N$

$\Updownarrow$

Maximize $g(\mu)$, where

dual

$$g(\mu) = \sum_{i=1}^{N} \inf_{x_i \in \mathcal{D}_i} \{f_i(x_i) - \mu_i^T x_i\} + \inf_{\tilde{x} \in \Re^{nN}} \{h(\tilde{x}) + \mu^T \tilde{x}\}$$

[Raffard, Boyd, Tomlin ‘04]

Summary

- The development of a reach set toolkit for hybrid systems:
  - Software C++; Matlab/QEPCAD
- The toolkit is useful for:
  - Engineering: when (not) to switch modes, which mode(s) to switch to, and provides a set-valued feedback control law to remain in safe set
  - Biology: can be used in reverse engineering of biological circuits, ie. used to extract ranges of parameters, determine feasibility of a particular influence connection – often as much system identification as modeling
- New algorithms, tools for (fast) analysis in high dimensions
  - Geometric approximation
  - Approximation algorithms based on optimization relaxations
  - Decentralization

Collaborators

Students and Former Students:
Ian Mitchell
Alex Bayen
Meeko Oishi
Rodney Teo
Jung Soon Jang
Inseok Hwang
Hamsa Balakrishnan
Robin Raffard
Ronojoy Ghosh
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