Discrete-Event Systems: Generalizing Metric Spaces and Fixed-Point Semantics

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Discrete-Event (DE) Systems

• Traditional Examples
  - VHDL
  - OPNET Modeler
  - NS-2

• Distributed systems
  - TeaTime protocol in Croquet

(two players vs. the computer)
Introduction to DE Systems

• In DE systems, concurrent objects (processes) interact via signals

What is the semantics of DE?

• Simultaneous events may occur in a model
  - VHDL Delta Time

• Simultaneity absent in traditional formalisms
  - Yates
  - Chandy/Misra
  - Zeigler
Time in Software

- Traditional programming language semantics lack time
- When a physical system interacts with software, how should we model time?
  - One possibility is to assume some computations take zero time, e.g.
    - Synchronous language semantics
    - GIOTTO logical execution time

Simultaneity in Hardware

- Simultaneity is common in synchronous circuits
- Example:

  ![Diagram of digital electronic circuit]

  This value changes instantly

  0 1 1
  0 1 1

  Time
Simultaneity in Physical Systems

Our Contributions

- We generalize DE semantics to handle simultaneous events

- We generalize metric space concepts to handle our model of time

- We give uniqueness conditions and conditions for avoidance of Zeno behavior
Models of Time

- Time (real time)

- Superdense time [Maler, Manna, Pnueli]

Zeno Signals

- Definition: Zeno Signal
  infinite events in finite real time

- Chattering Zeno [Ames]

- Genuinely Zeno [Ames]
Source of Zeno Signals

- Feedback can cause Zeno

Genuinely Zeno

- A source of genuinely Zeno signals
Simple Processes

- Definition: *Simple Process*

\[ \text{Process} \rightarrow \text{Non-Zeno Signal} \]

- Merge is simple, but it has Zeno feedback solutions

\[ \text{Merge} \rightarrow \text{Non-Zeno Signal} \]

- When are compositions of simple processes simple?

Cantor Metric for Signals

"Distance" between two signals

\[ d(s_1, s_2) = \frac{1}{2^t} \]

First time at which the two signals differ

\[ \text{First time at which the two signals differ} \]
Tetrics: Extending Metric Spaces

- Cantor metric doesn't capture simultaneity
- Tetrics are generalized metrics
- We generalized metric spaces with “tetric spaces”
- Our tetric allows us to deal with simultaneity

Our Tetric for signals

"Distance" between two signals:

\[ d(s_1, s_2) = \left( \frac{1}{2^t}, \frac{1}{2^n} \right) \]

\[ d_1(s_1, s_2) = \frac{1}{2^t} \]

First time at which the two signals differ

\[ d_2(s_1, s_2) = \frac{1}{2^n} \]

First sequence number at which the two signals differ
**Delta Causal**

Definition: *Delta Causal*
Input signals agree up to time $t$ implies
output signals agree up to time $t + \Delta$

![Diagram of Delta Causal Process]

**What Delta Causal Means**

- Signals which delay their response to input events by delta will have non-Zeno fixed points

![Diagram of Delta Causal Process with time points]
Extending Delta Causal

- The system should be allowed to chatter

\[ \text{Delta Causal Process} \]

- As long as time eventually advances by delta

Tetric Delta Causal

Definition: Tetric Delta Causal

1) Input signals agree up to time \((t, n)\) implies output signals agree up to time \((t, n + 1)\)

2) If \(n\) is large enough, this also implies output signals agree up to time \((t + \Delta, 0)\)
**Causal**

Definition: *Causal*
If input signals agree up to supertime \((t, n)\) then the output signals agree up to supertime \((t, n)\)

**Result 1**

- Every (extended) delta causal process has a unique feedback solution
Result 2

- Every network of simple, causal processes is a simple causal process, provided in each cycle there is a delta causal process.
- Example

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<table>
<thead>
<tr>
<th>Delay</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Merge</td>
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<tr>
<td>Non-Zeno Input Signal</td>
</tr>
</tbody>
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Conclusions

- We broadened DE semantics to handle superdense time.
- We invented tetric spaces to measure the distance between DE signals.
- We gave conditions under which systems will have unique fixed-point solutions.
- We provided sufficient conditions under which this solution is non-Zeno.
- http://ptolemy.eecs.berkeley.edu/papers/05/DE_Systems
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