Stochastic Zero-sum and Nonzero-sum ω-regular Games

A Survey of Results

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Chess Review

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Outline

1. Stochastic games: informal descriptions.
2. Classes of game graphs.
3. Objectives.
4. Strategies.
5. Outline of results.
6. Open Problems.

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Stochastic Games

- Games played on game graphs with stochastic transitions.
- Stochastic games [Sha53]
  - Framework to model natural interaction between components and agents.
    - e.g., controller vs. system.

Stochastic Games

- Where:
  - Arena: Game graphs.
- What for:
  - Objectives - ω-regular.
- How:
  - Strategies.

Game Graphs

- Two broad class:
  - Turn-based games
    - Players make moves in turns.
  - Concurrent games
    - Players make moves simultaneously and independently.

Classification of Games

- Games can be classified in two broad categories:
  - Zero-sum games:
    - Strictly competitive, e.g., Matrix games.
  - Nonzero-sum games:
    - Not strictly competitive, e.g., Bimatrix games.

Goals

- Determinacy: minmax and maxmin values for zero-sum games.
- Equilibrium: existence of equilibrium payoff for nonzero-sum games.

- Computation issues.
- Strategy classification: simplest class of strategies that suffice for determinacy and equilibrium.

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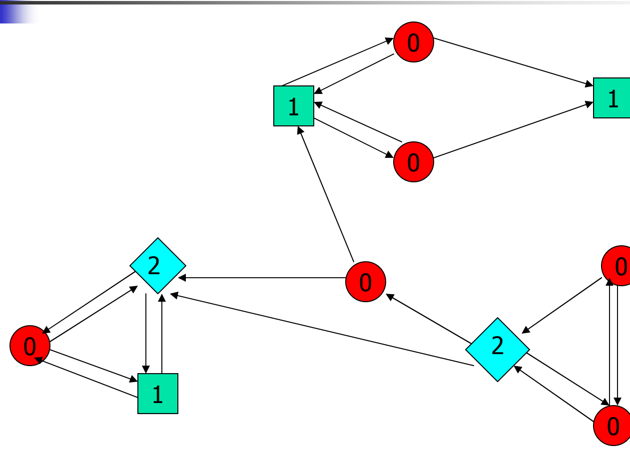
Turn-based Games

Turn-based Probabilistic Games

- A turn-based probabilistic game is defined as
  - $G=(V,E,(V_1,V_2,V_0))$, where
    - (V,E) is a graph.
    - (V_1,V_2,V_0) is a partition of V.
- V_1 player 1 makes moves.
- V_2 player 2 makes moves.
- V_0 randomly chooses successors.

A Turn-based Probabilistic Game



Special Cases

- Turn-based deterministic games:
  - $V_0 = \phi$ (emptyset).
  - No randomness, deterministic transition.
- Markov decision processes (MDPs)
  - $V_2 = \phi$ (emptyset).
  - No adversary.

Applications

- MDPs (1 ½- player games)
  - Control in presence of uncertainty.
  - Games against nature.
- Turn-based deterministic games (2-player games)
  - Control in presence of adversary, control in open environment or controller synthesis.
  - Games against adversary.
- Turn-based stochastic games (2 ½ -player games)
  - Control in presence of adversary and nature, controller synthesis of stochastic reactive systems.
  - Games against adversary and nature.

Game played

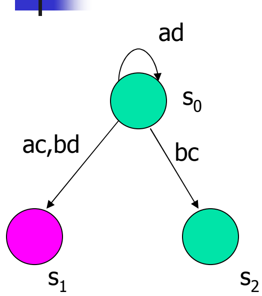
- Token placed on an initial vertex.
- If current vertex is
  - Player 1 vertex then player 1 chooses successor.
  - Player 2 vertex then player 2 chooses successor.
  - Player random vertex proceed to successors uniformly at random.
- Generates infinite sequence of vertices.

Concurrent Games

Concurrent game

- Players make move simultaneously.
- Finite set of states S.
- Finite set of actions Σ.
- Action assignments
  - $\Gamma_1, \Gamma_2 : S \to 2^{\Sigma} \setminus \phi$
- Probabilistic transition function
  - $\delta(s, a_1, a_2)(t) = \Pr[\, t \mid s, a_1, a_2]$

Concurrent game

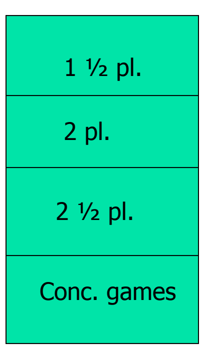


Actions at s_0: a, b for player 1,
c, d for player 2.

Concurrent games

- Games with simultaneous interaction.
- Model synchronous interaction.

Stochastic games



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Objectives

Plays

- Plays: infinite sequence of vertices or infinite trajectories.
- V^ω: set of all infinite plays or infinite trajectories.

Objectives

- Plays: infinite sequence of vertices.
- Objectives: subset of plays, $\Psi_1 \subseteq V^\omega$.
- Play is winning for player 1 if it is in Ψ_1
- Zero-sum game: $\Psi_2 = V^\omega \setminus \Psi_{1.}$

Reachability and Safety

- Let $R \subseteq V$ set of target vertices. Reachability objective requires to visit the set R of vertices.

- Let $S \subseteq V$ set of safe vertices. Safety objective requires never to visit any vertex outside S.

Buchi Objective

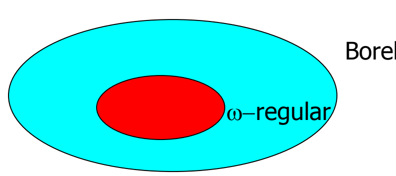
- Let $B \subseteq V$ a set of Buchi vertices.
  Buchi objective requires that the set B is visited infinitely often.

Rabin-Streett

- Let $\{(E_1,F_1), (E_2,F_2),\ldots, (E_d,F_d)\}$ set of vertex set pairs.
  - Rabin: requires there is a pair (E_j,F_j) such that E_j finitely often and F_j infinitely often.
  - Streett: requires for every pair (E_j,F_j) if F_j infinitely often then E_j infinitely often.
  - Rabin-chain: both a Rabin-Streett, complementation closed subset of Rabin.

Objectives

- ω-regular: $\cup$, ° , *,ω.
  - Safety, Reachability, Liveness, etc.
  - Rabin and Streett canonical ways to express.



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Strategies

Strategy

- Given a finite sequence of vertices, (that represents the history of play) a strategy σ for player 1 is a probability distribution over the set of successor.
  - $\sigma : V^* \cdot V_1 \rightarrow D$

Subclass of Strategies

- Memoryless (stationary) strategies: Strategy is independent of the history of the play and depends on the current vertex.
  - $\sigma: V_1 \rightarrow D$
- Pure strategies: chooses a successor rather than a probability distribution.
- Pure-memoryless: both pure and memoryless (simplest class).

Strategies

- The set of strategies:
  - Set of strategy Σ for player 1; strategies σ.
  - Set of strategy Π for player 2; strategies π.

Values

- Given objectives Ψ_1 and $\Psi_2 = V^\omega \setminus \Psi_1$ the value for the players are
  - $v_1(\Psi_1)(v) = \sup_{\sigma \in \Sigma} \inf_{\pi \in \Pi} Pr_v^{\sigma,\pi}(\Psi_1)$.
  - $v_2(\Psi_2)(v) = \sup_{\pi \in \Pi} \inf_{\sigma \in \Sigma} Pr_v^{\sigma,\pi}(\Psi_2)$.

Determinacy

- Determinacy: $v_1(\Psi_1)(v) + v_2(\Psi_2)(v) = 1$.
- Determinacy means
  - sup inf = inf sup.
  - von Neumann's minmax theorem in matrix games.

Optimal strategies

- A strategy σ is optimal for objective Ψ_1 if
  - $v_1(\Psi_1)(v) = \inf_\pi \Pr_v^{\sigma,\pi}(\Psi_1)$.
- Analogous definition for player 2.

Zero-sum and nonzero-sum games

- Zero sum: $\Psi_2 = V^\omega \setminus \Psi_1$.
- Nonzero-sum: Ψ_1 and Ψ_2
  - happy with own goals.

Concept of rationality

- Zero sum game: Determinacy.
- Nonzero sum game: Nash equilibrium.

Nash Equilibrium

- A pair of strategies (π_1, π_2) is an ε-Nash equilibrium if
  - For all π'_1, π'_2:
    - $\text{Value}_2(\pi_1, \pi'_2) \leq \text{Value}_2(\pi_1, \pi_2) + \varepsilon$
    - $\text{Value}_1(\pi'_1, \pi_2) \leq \text{Value}_1(\pi_1, \pi_2) + \varepsilon$
  - Neither player has advantage of more than ε in deviating from the equilibrium strategy.
  - A 0-Nash equilibrium is called a Nash equilibrium.
  - Nash's Theorem guarantees existence of Nash equilibrium in nonzero-sum matrix games.

Computational Issues

- Algorithms to compute values in games.
- Identify the simplest class of strategies that suffices for optimality or equilibrium.

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Outline of results

History and results

- MDPs
  - Complexity of MDPs. [PapTsi89]
  - MDPs with ω-regular objectives. [CouYan95,deAl97]

History and results

- Two-player games.
  - Determinacy (sup inf = inf sup) theorem for Borel objectives. [Mar75]
  - Finite memory determinacy (i.e., finite memory optimal strategy exists) for ω-regular objectives. [GurHar82]
  - Pure memoryless optimal strategy exists for Rabin objectives. [EmeJut88]
    - NP-complete.

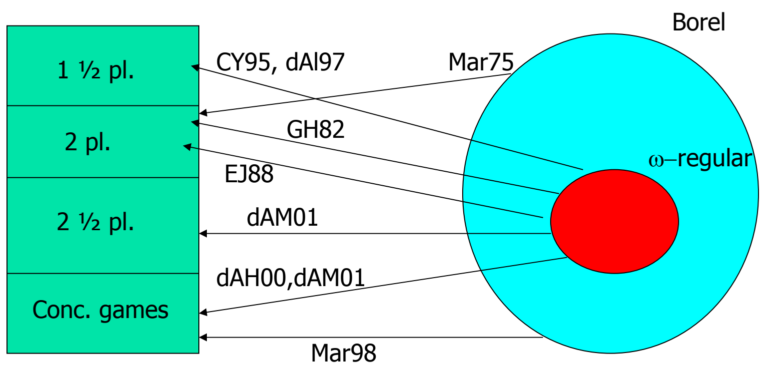
History and result

- 2 ½ - player games
  - Reachability objectives: [Con92]
    - Pure memoryless optimal strategy exists.
    - Decided in NP $\cap$ coNP.

History and results: Concurrent zero-sum games

- Detailed analysis of concurrent games [FilVri97].
- Determinacy theorem for all Borel objectives [Mar98].
- Concurrent ω-regular games:
  - Reachability objectives [deAlHenKup98].
  - Rabin-chain objectives [deAlHen00].
  - Rabin-chain objectives [deAlMaj01].

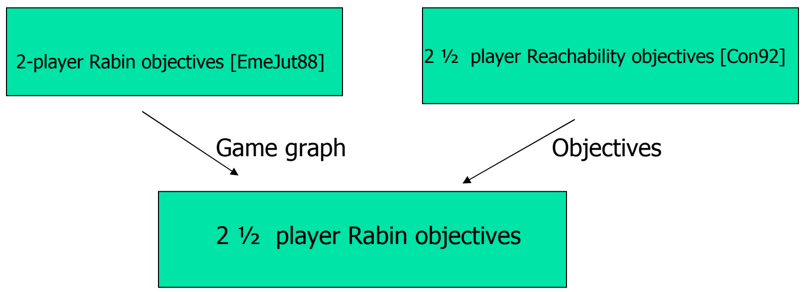
Zero sum games



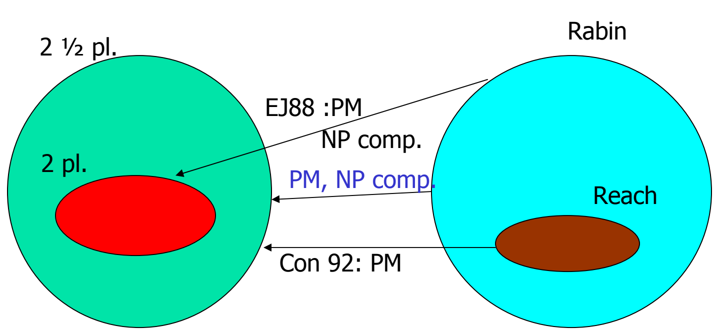
Zero sum games

- 2 ½ player games with Rabin and Streett objectives [CdeAlHen 05a]
  - Pure memoryless optimal strategies exist for Rabin objectives in 2 ½ player games.
  - 2 ½ player games with Rabin objectives is NP-complete.
  - 2 ½ player games with Streett objectives is coNP-complete.

Zero sum games



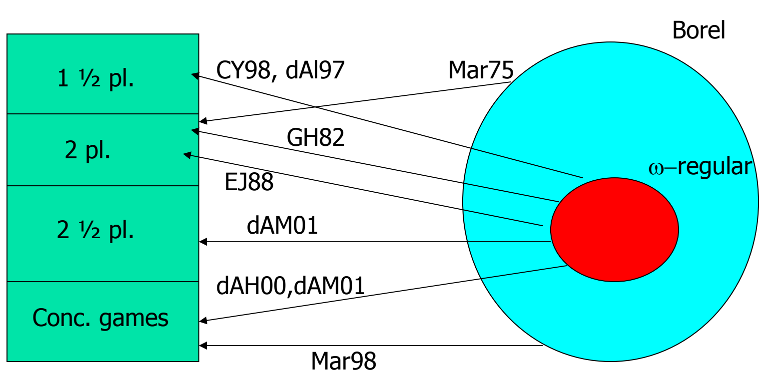
Zero-sum games



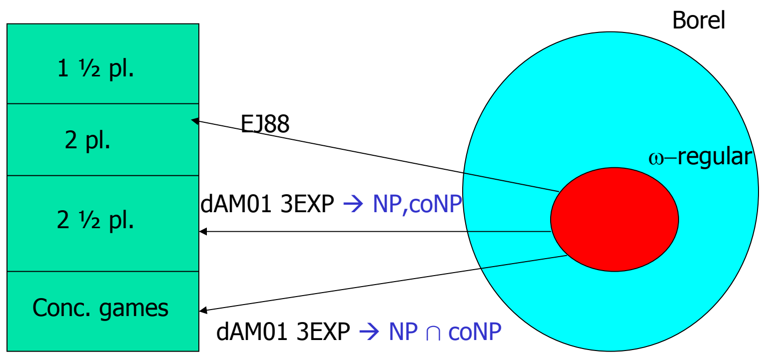
Zero sum games

- Concurrent games with parity objectives
  - Requires infinite memory strategies even for Buchi objectives [deAlHen00].
  - Polynomial witnesses for infinite memory strategies and polynomial time verification procedure.
  - Complexity: NP ∩ coNP [CdeAlHen 05b].

Zero sum games



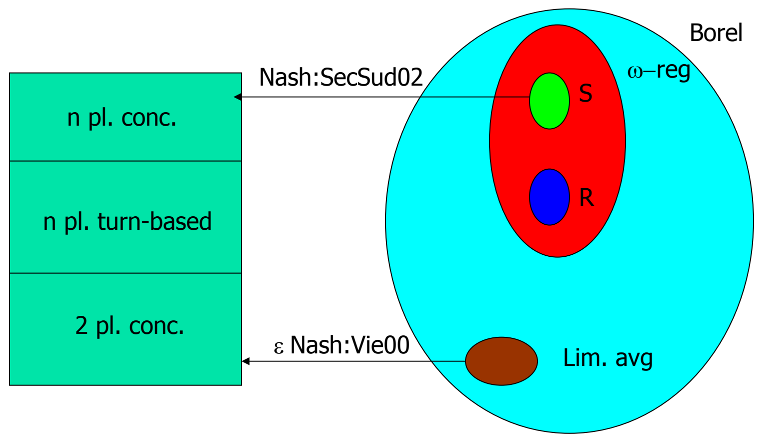
Zero sum games



History: Nonzero-sum Games

- Two-player nonzero-sum stochastic games with limit-average payoff. [Vie00a, Vie00b]
- Closed sets (Safety). [SecSud02]

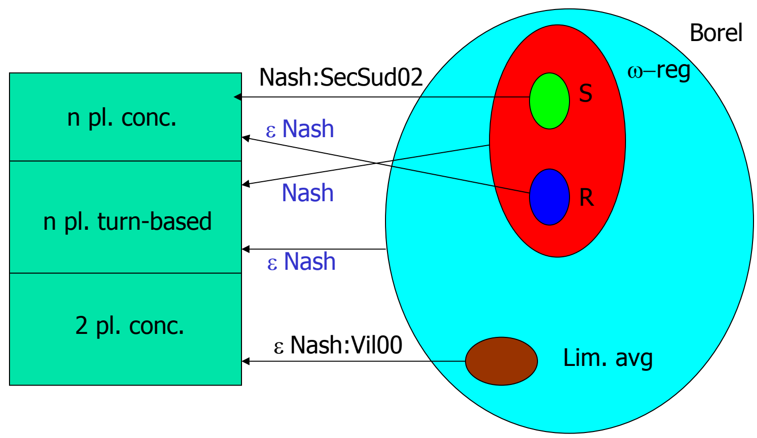
Nonzero sum games



Nonzero sum games

- For all n player concurrent games with reachability objectives for all players, ε-Nash equilibrium exist for all ε >0, in memoryless strategies [CMajJur 04].
- For all n player turn-based stochastic games with Borel objectives for the players, ε-Nash equilibrium exist for all ε >0, in pure strategies [CMajJur 04].
  - The result strengthens to exact Nash equilibria in case of n player turn based deterministic games with Borel objectives, and n player turn based stochastic games with ω-regular objectives.

Nonzero sum games

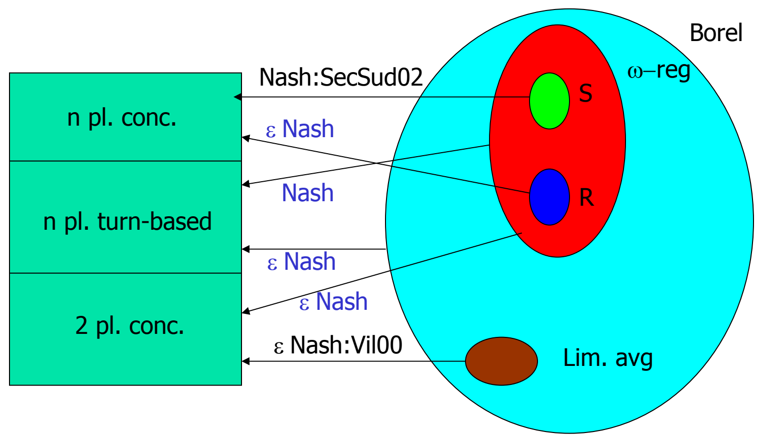


Nonzero sum games

- For 2-player concurrent games with ω-regular objectives for both players, ε-Nash equilibrium exist for all ε >0 [C 05].

- Polynomial witness and polynomial time verification procedure to compute an ε-Nash equilibrium.

Nonzero sum games



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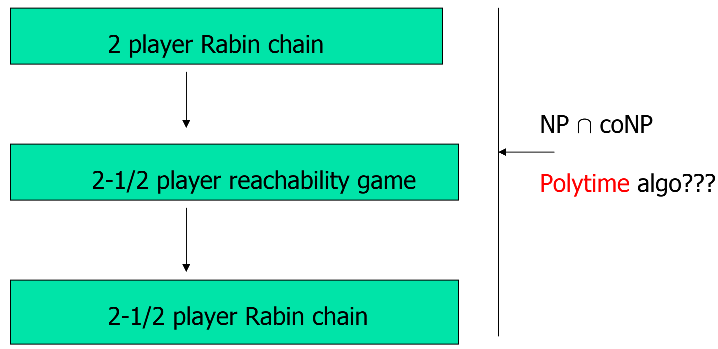
Major open problems



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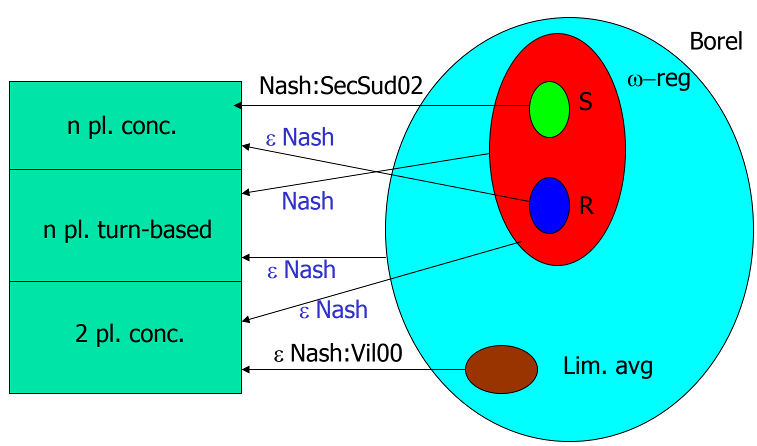
Nonzero sum games



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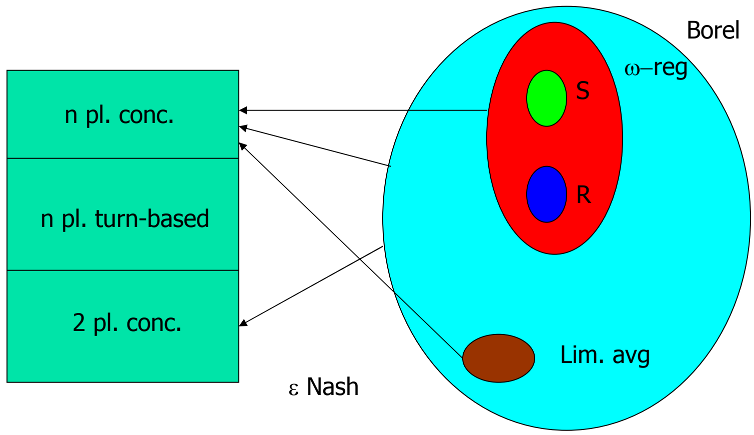
Nonzero sum games



Conclusion

- Stochastic games
  - Rich theory.
  - Communities: Descriptive Set Theory, Stochastic Game Theory, Probability Theory, Control Theory, Optimization Theory, Complexity Theory, Formal Verification … .
  - Several open theoretical problems.

Joint work with

Thomas A. Henzinger
Luca de Alfaro
Rupak Majumdar
Marcin Jurdzinski

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Thanks !!!

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