Operational Semantics of Hybrid Systems

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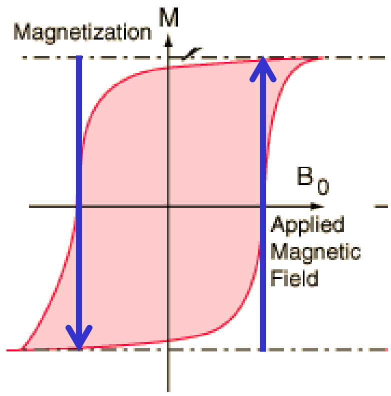
The Objective

- Hybrid systems can be treated as executable programs written in a domain-specific programming language.
- We need an operational semantics to execute these programs to generate behaviors of hybrid systems.

A Hysteresis Example as a Hybrid System

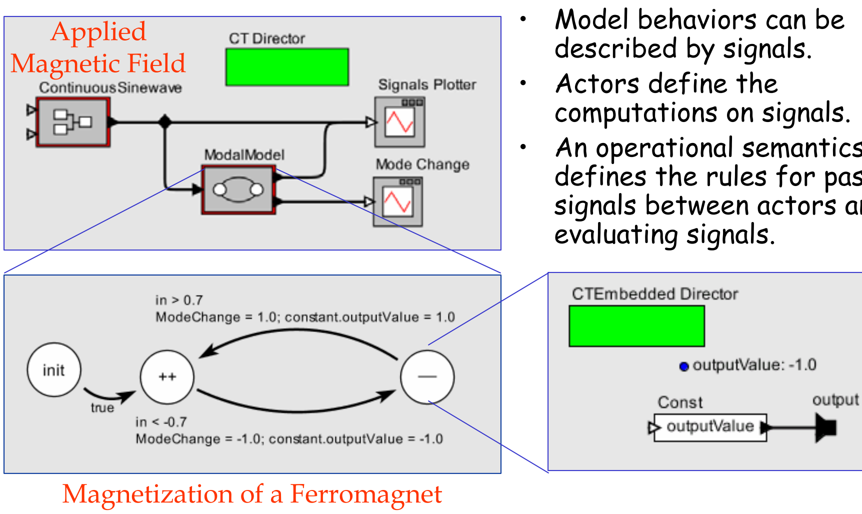


The magnetization of a ferromagnet depends on not only the current magnetic field strength but also the history magnetic flux density.

The magnetization process can be abstracted as a instantaneous change if the duration is neglectable.

Borrowed and modified from:
http://hyperphysics.phy-astr.gsu.edu/hbase/solids/hyst.html

Hybrid Systems as Executable Programs

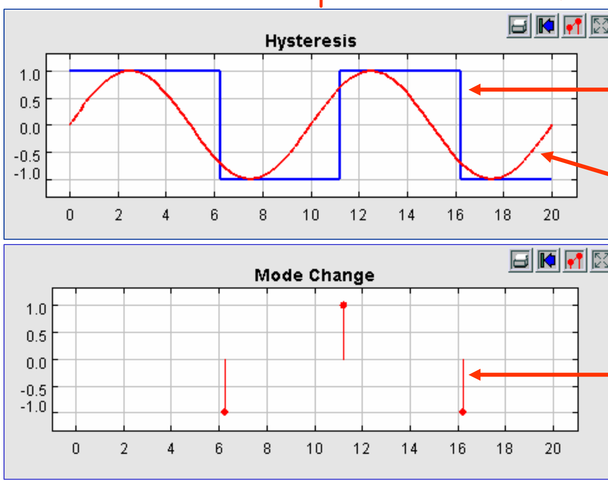


- Model behaviors can be described by signals.
- Actors define the computations on signals.
- An operational semantics defines the rules for passing signals between actors and evaluating signals.

This model is constructed with HyVisual, which is based on Ptolemy II.

Signals in the Hysteresis Model

- Discontinuities are caused by discrete events.
- At discontinuities, signals have multiple values, which are in a particular order.

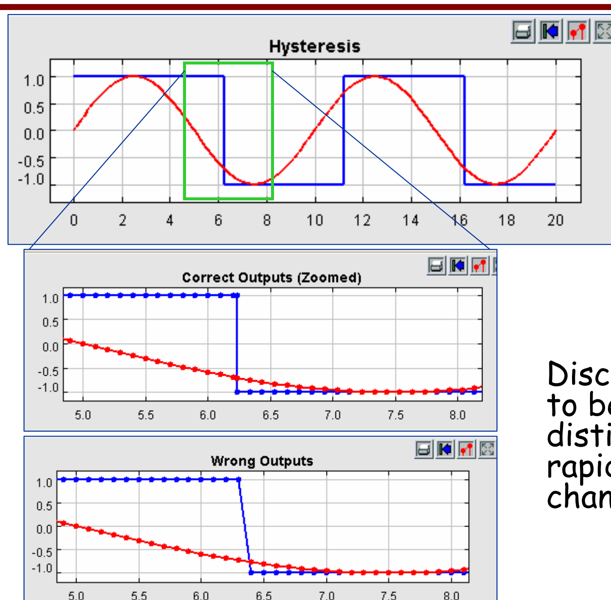


piecewise continuous signal (magnetic flux density)

continuous signal (applied magnetic field)

discrete-event signal (mode changes)

Signals Must Have Multiple Values at the Time of a Discontinuity



Discontinuities need to be semantically distinguishable from rapid continuous changes.

Definition: *Continuously Evolving Signal*

- Define a continuously evolving signal as the function:

$$x: T \times \mathbb{N} \rightarrow V$$

- Where T is a connected subset of reals (time line) and N is the set of non-negative integers (indexes).

  The domain is also called "super dense time" by Oded Maler, Zohar Manna, and Amir Pnueli in "From timed to hybrid systems", 1992.

- At each time $t \in T$, the signal x has a sequence of values. Where the signal is continuous, all the values are the same. Where is discontinuous, it has multiple values.

Zeno Signals

- Chattering Zeno signal
  - There exists a time $t \in T$, where the signal x has an infinite sequence of *different* values.
- Genuinely Zeno signal
  - There exists a time $t \in T$, where the signal x has an infinite number of discontinuities before t.
- Zeno signals cause problems in simulation.
  - The simulation time cannot progress over t before the infinite number of different values of a chatting Zeno signal at t are resolved.
  - The simulation time cannot even reach t before the values at the infinite number of times where discontinuities happen are resolved.

Initial and Final Value Signals

- A signal $x: T \times \mathbb{N} \to V$ has no *chattering Zeno* condition if there is an integer $m >= 0$ such that
  $$\forall n > m, \quad x(t,n) = x(t,m)$$
- A non-chattering signal has a corresponding *final value signal,* $x_f: T \to V$ where
  $$\forall\, t \in T, \quad x_f(t) = x(t,m)$$
- A non-chattering signal has a corresponding *initial value signal,* $x_i: T \to V$ where
  $$\forall\, t \in T, \quad x_i(t) = x(t,0)$$

Revisiting Signals in Hybrid Systems

- For a signal x, define $D \subset T$ as the set that contains all time points where discontinuities of x happen.
- A piecewise continuous signal is a non-chattering signal
  $$x: T \times \mathbb{N} \to V$$
  - where:
    the initial value signal x_i is continuous on the left,
    the final value signal x_f is continuous on the right, and
    the signal x has only one value at all $t \in T \setminus D$.
- A continuous signal is a piecewise continuous signal where D is an empty set.
- A discrete-event signal is a continuously evolving signal that does not have a value almost everywhere except at the time points included in D.

Discrete Trace

- Define a *discrete trace* of a signal x as a set:

  $\{ x(t, n) \mid t \in D', \text{ and } n \in N \}$

  where $D' = D \bigcup \{t_0\} \subset T$

  and D is the set of time points where x has discontinuities and t_0 is the starting time when the signal x is defined.
- Discrete traces capture all the details of signals, hence the behavior of hybrid systems.
  - The *ideal solver semantics* in "On the causality of mixed-signal and hybrid models" by Jie Liu and Edward A. Lee, HSCC 2003

Operational Semantics

An execution of a hybrid system is the process of constructing discrete traces for all signals.

Constructing Discrete Traces

- A discrete trace of a signal x is constructed by a sequence of *unit* executions.
- A unit execution is defined on a time interval $[t_1, t_2)$ for each neighboring $t_1 < t_2 \in D'$
- Each unit execution consists of two phases of executions in the following order:
  - Discrete phase of execution:
    Construct the discrete trace of signal x at t_1
  - Continuous phase of execution:
    Find the initial value of signal x at t_2

Discrete Phase of Execution

- Given $x_i(t_1) = x(t_1, 0)$, perform

$$x(t_1,1) = f(x(t_1,0), t_1)$$
$$x(t_1,2) = f(x(t_1,1), t_1)$$
$$x(t_1,3) = f(x(t_1,2), t_1)$$
$$\dots$$

till the final value of x, $x_f(t_1)$, is reached.

- The subset of the discrete trace of signal x at t_1

$$\{ x(t_1, n) \mid t_1 \in D', \text{and } n \in N \}$$

is completely resolved.

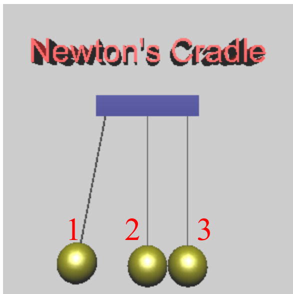
Continuous Phase of Execution

- Given $x_f(t_1)$ and the ordinary differential equations governing the dynamics of signal x satisfying a *global Lipschitz* condition during the time interval (t_1, t_2), there exists a unique solution for x and it can be approximated by ODE solvers.
- The value of signal x at any $t \in T$ can be computed, where $t_1 < t < t_2$.
- The discrete trace loses nothing by not representing values within the interval.

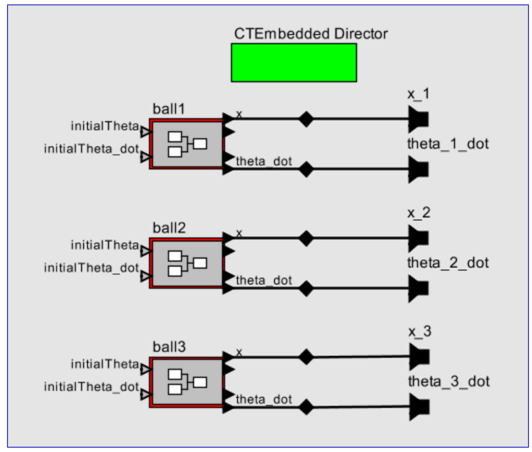
Importance of Discrete Phase Of Execution

- Having multiple events at the same time is a useful abstraction for modeling software, hardware, and physical phenomena.
- With the discrete phase of execution
  - This operational semantics handles these simultaneous events by giving them a well defined order.
  - This operational semantics completely captures the model's behavior at discontinuities.

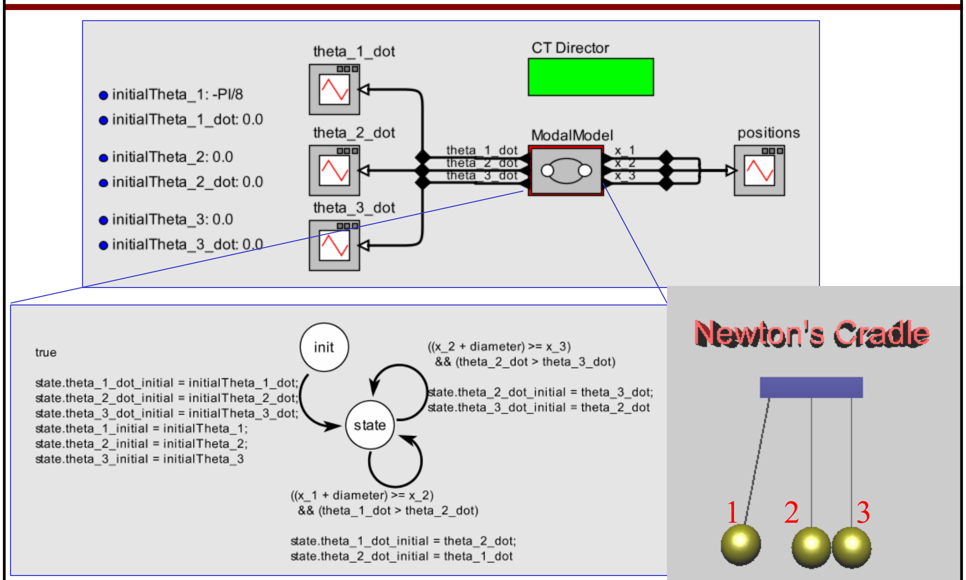
Newton's Cradle: Dynamics



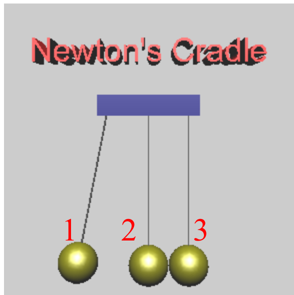
Three second order ODE's are used to model the dynamics of three pendulums.



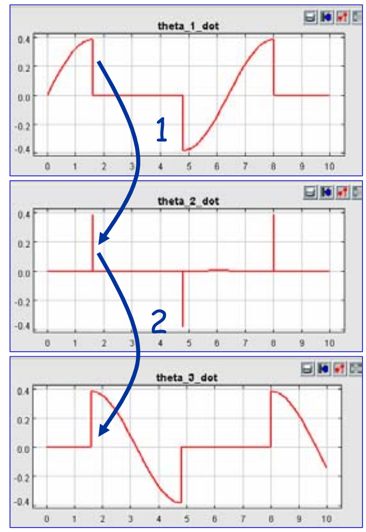
Newton's Cradle: Mode Control

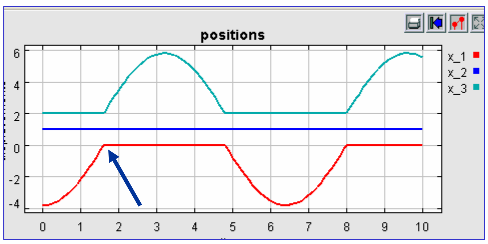


Velocities and Positions of Balls



X-axis is time and Y-axis is velocity.





X-axis is time and Y-axis is displacement.

Summary

- Signals in hybrid systems are studied and defined with the "super dense time" as domain.

- An operational semantics to construct discrete traces of signals in hybrid system is given.

  For more details, check "Operational Semantics of Hybrid Systems" by Edward A. Lee and Haiyang Zheng, HSCC 2005.

- This operational semantics is implemented in HyVisual 5.0 and Ptolemy II 5.0, which can be downloaded from http://ptolemy.eecs.berkeley.edu/ptolemyII.