Closing the loop around Sensor Networks

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Conceptual Issues

- Given a certain wireless sensor network can we successfully design a particular application?
- How does the application impose constraints on the network?
- Can we derive important metrics from those constraints?
- How do we measure network parameters?

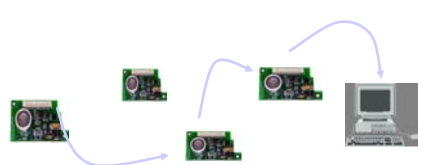
What can you do with a sensor network?

- Literature provides key asymptotic results
- We are interested in answering different semantic questions, e.g.:
- At the algorithmic level:
  - How much packet loss can a tracking algorithm tolerate?
- At the network level:
  - How many objects can a particular sensor network reliably track?

Wireless Sensor Networks

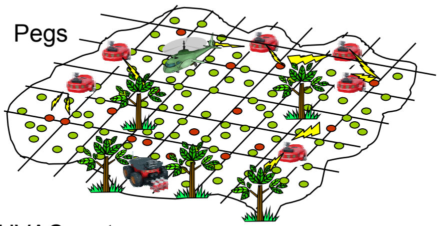
- It's a network of devices:
  - Many nodes: 10^3-10^5
  - Multi-hop wireless communication with adjacent nodes
  - Cheap sensors
  - Cheap CPU

- Issues w/ Sensor Networks and Data Networks ?
  - Random time delay
  - Random arrival sequence
  - Packet loss
  - Limited Bandwidth

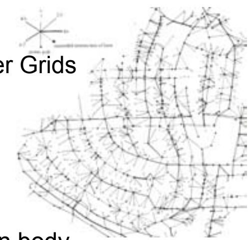


Control Applications with Sensor Networks

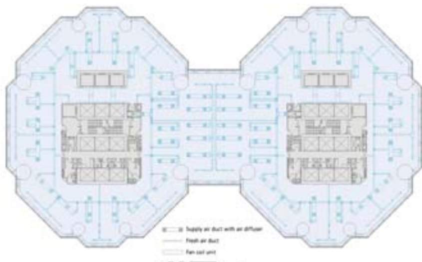
Pegs



Power Grids

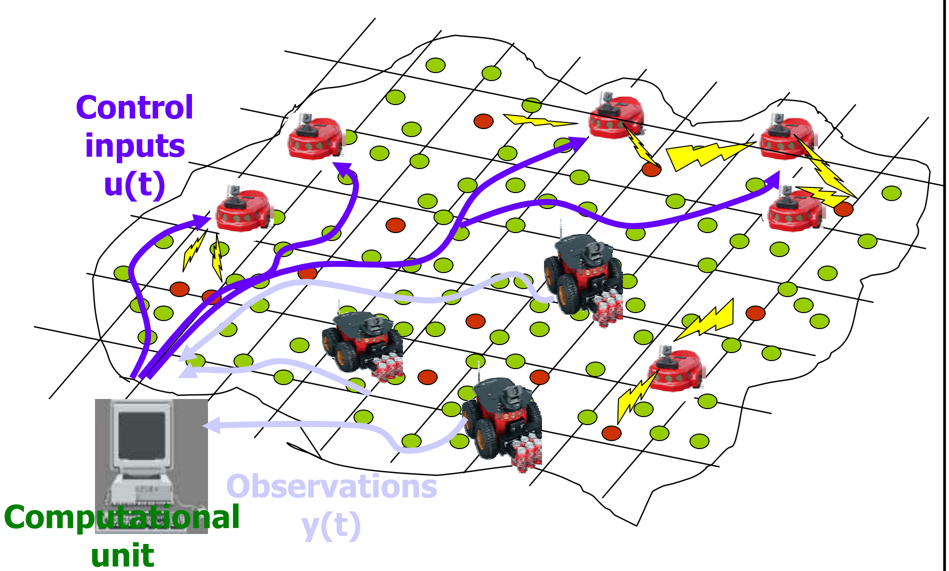


HVAC systems



Human body

Control and communication over Sensor Networks



Experimental results: Pursuit evasion games

Problem Statement

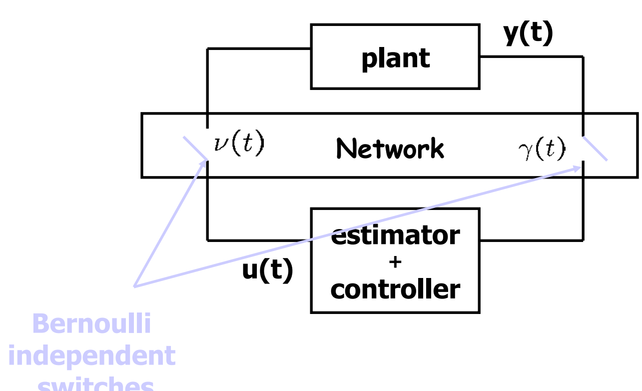
Given a control systems where components, i.e. plant, sensors, controllers, actuators, are connected via a specified communication network, design an "optimal" controller for the system

Outline

- Problem Statement
- Optimal Estimation with intermittent observations
- Optimal control with both intermittent obs and control
  - TCP-like protocols
  - UDP-like protocols
- Conclusions

Modeling

LGQ scenario for lossy network



Assumptions

- System:
  - Discrete time linear time invariant
  - Additive white gaussian noise on both the dynamics and the observation
- Communication network:
  - Packets either arrive or are lost within a sampling period following a bernoulli process.
  - A Delay longer than sampling time is considered lost.
  - Packet Acknowledgement depends on the specific communication protocol

Optimal estimation with intermittent observations

$$x_{t+1} = Ax_t + w_t$$
$$y_t = Cx_t + v_t,$$

$p_{\gamma_t}(1) = \bar{\gamma}$

Kalman Filter

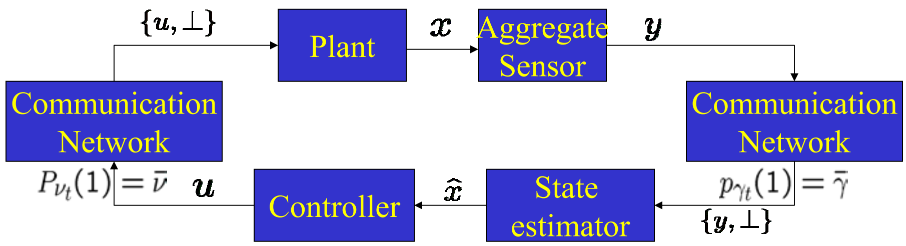
- Main Results (IEEE TAC September 2004)
- Kalman Filter is still the optimal estimator
- We proved the existence of a threshold phenomenon:

$$\lim_{t\to\infty} E[P_t] = \infty \quad \text{for } 0 \le \bar{\gamma} \le \gamma_c \text{ and some initial condition } P_0 \ge 0$$

$$E[P_t] \le M_{P_0} \ \forall t \text{ for } \gamma_c < \bar{\gamma} \le 1 \text{ and any initial condition } P_0 \ge 0$$

$$1 - \frac{1}{(|\lambda_{max}|)^2} = \gamma_{min} \le \gamma_c \le \gamma_{max}$$

Optimal control with both intermittent observations and control packets

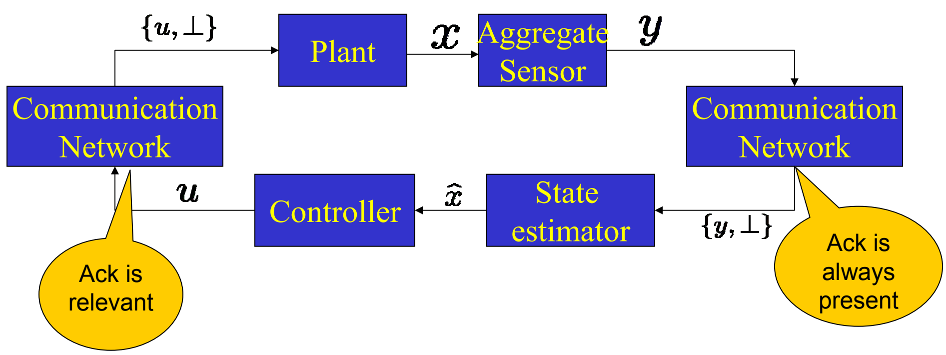


- What is the minimum arrival probability that guarantees "acceptable" performance of estimator and controller?
- How is the arrival rate related to the system dynamics?
- Can we design estimator and controller independently?
- Are the optimal estimator and controllers still linear?
- Can we provide design guidelines?

Control approach

- The problem of control is traditionally subdivided in two sub-problems:
  - Estimation
    - Allows to recover state information from observations
  - Control
    - Given current state information, control inputs are provided to the actuators
- The separation principle:
  - allows, under observability conditions, to design estimator and controller independently.
- If separation principle holds, optimal estimator (in the minimum variance sense) and optimal controller (LQG) are linear and independent

LQG control with intermittent observations and control



We'll group all communication protocols in two classes:
TCP-like (acknowledgement is available)
UDP-like (acknowledgement is absent)

LQG mathematical modeling

$$x_{t+1} = Ax_t + \nu_t B u_t + w_t$$
$$y_t = Cx_t + v_t, \qquad \text{cov}(v_t) = \gamma_t R + (1-\gamma_t)\sigma^2 I$$
$$J_N = \mathbb{E}[x_N' W_N x_N + \textstyle\sum_{t=0}^{N-1}(x_t' W_t x_t + \nu_t\, u_t' U_t u_t)]$$

γ,ν – Bernoulli, indep.

Minimize J_N subject to

$$u_t = k_t(\mathcal{I}_t); \quad \mathcal{I}_t = \begin{cases} y_0, \ldots, y_t, \gamma_0, \ldots, \gamma_t, \nu_0, \ldots, \nu_{t-1} & TCP \\ y_0, \ldots, y_t, \gamma_0, \ldots, \gamma_t & UDP \end{cases}$$

- **TCP – Transmission Control Protocol**
  - PRO: feedback information on packet delivery
  - CONS: more expensive to implement
- **UDP – User Datagram Protocol**
  - PRO: simpler communication infrastructure
  - CONS: less information available

Estimator Design

TCP **UDP**

Prediction Step

TCP:

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t} + \nu_t B u_t$$
$$P_{t+1|t} = AP_{t|t}A' + Q$$

UDP:

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t} + \bar{\nu} B u_t$$
$$P_{t+1|t} = AP_{t|t}A' + Q + \bar{\nu}(1-\bar{\nu})Bu_t u_t' B$$

Correction Step

$$\begin{aligned} \hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} + \gamma_{t+1} K_{t+1}(y - C\hat{x}_{t+1|t}) \\ P_{t+1|t+1} &= P_{t+1|t} - \gamma_{t+1} P_{t+1|t} C'(CP_{t+1|t}C' + R)^{-1} C P_{t+1|t} \\ K_{t+1} &= P_{t+1|t} C'(CP_{t+1|t}C' + R)^{-1} \end{aligned}$$

LQG Controller Design: TCP case

Solution via Dynamic Programming:

1. **Compute the Value Function t=N and move backward**
2. **Find Infinite Horizon by taking N→+1**

$V_t(x_t)$ – minimum cost-to-go if in state x_t at time t

$$V_N(x_N) \triangleq \mathbb{E}[x_N' W_N x_N \mid \mathcal{F}_N]$$

$$V_k(x_k) \triangleq \min_{u_k} \mathbb{E}[x_k' W_k x_k + \nu_k u_k' U_k u_k + V_{k+1}(x_{k+1}) \mid \mathcal{F}_k]$$

$$J_N = \mathbb{E}[x_N' W_N x_N + \textstyle\sum_{t=0}^{N-1}(x_t' W_t x_t + \nu_t\, u_t' U_t u_t)]$$

$$J_N^* = V_0(x_0)$$

LQG Controller Design: TCP case

We can prove that for TCP the value function can be written as:

$$V_t(x_t) = \mathbb{E}[x_t' S_t x_t \,|\, \mathcal{I}_t] + c_t$$

with:

$$S_N = W_N, \qquad c_N = 0$$

$$\begin{aligned} S_t &= A'S_{t+1}A + W_t - \bar{\nu}A'S_{t+1}B(W_t + B'S_{t+1}B)^{-1}BS_{t+1}A \\ c_t &= \mathbb{E}[c_{t+1} \,|\, \mathcal{I}_t] + \mathrm{trace}[(A'S_{t+1}A + W_t - S_t)P_{t|t}] + \mathrm{trace}(S_{t+1}Q) \end{aligned}$$

Minimization of v(t) yields:

$$\begin{aligned} u_t &= -(W_t + B'S_{t+1}B)^{-1}B'S_{t+1}A\hat{x}_{t|t} \\ &= -L_k\hat{x}_{t|t} \end{aligned}$$

LQG Controller Design: TCP case

$$J_N^* = x_0'S_0x_0 + \mathrm{trace}(S_0P_0) + \sum_{t=0}^{N-1}\left(\mathrm{trace}\left((A'S_{t+1}A + W_t - S_t)\mathbb{E}_\gamma[P_{t|t}]\right) + \mathrm{trace}(S_{t+1}Q)\right)$$

Stochastic variable !!

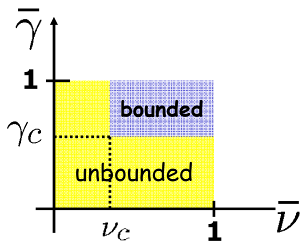
$$\tilde{P}_{t|t} \leq \mathbb{E}_\gamma[P_{t|t}] \leq \hat{P}_{t|t}$$

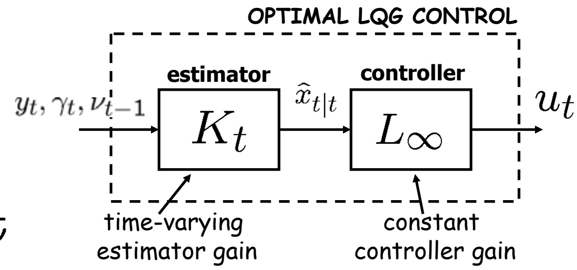
$$\begin{aligned} \hat{P}_{t+1|t} &= A\hat{P}_{t|t-1}A' + Q - \bar{\gamma}A\hat{P}_{t|t-1}C'(C\hat{P}_{t|t-1}C' + R)^{-1}C\hat{P}_{t|t-1}A' \\ \hat{P}_{t|t} &= \hat{P}_{t|t-1} - \bar{\gamma}\hat{P}_{t|t-1}C'(C\hat{P}_{t|t-1}C' + R)^{-1}C\hat{P}_{t|t-1} \\ \tilde{P}_{t+1|t} &= (1-\bar{\gamma})A\tilde{P}_{t|t-1}A' + Q \\ \tilde{P}_{t|t} &= (1-\bar{\gamma})\tilde{P}_{t|t-1} \end{aligned}$$

Infinite Horizon: TCP case

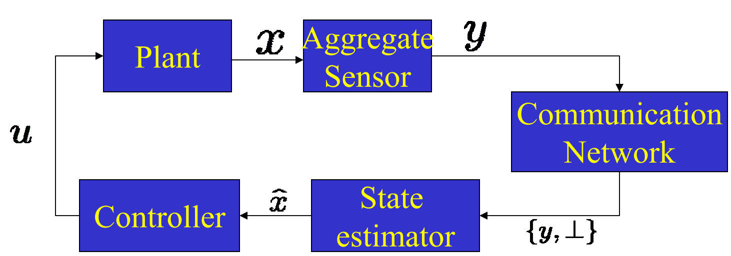
LQG averaged cost $\frac{J_N^*}{N}$ is bounded for all N if the following Modified Algebraic Riccati Equations exist:

$$
\begin{aligned}
S_\infty &= A'S_\infty A + W_t - \bar{\nu}\overbrace{A'S_\infty B(W_t + B'S_\infty B)^{-1}BS_\infty A}^{L_\infty} \\
\hat{P}_\infty &= A\hat{P}_\infty A' + Q - \bar{\gamma}A\hat{P}_\infty C'(C\hat{P}_\infty C' + R)^{-1}C\hat{P}_\infty A'
\end{aligned}
$$



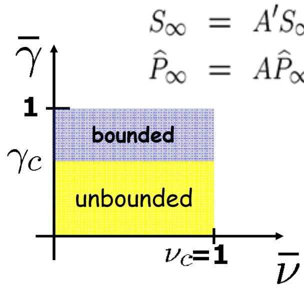


Special Case: LQG with intermittent observations, $\nu_t = 1, \forall t$



$$
\begin{aligned}
S_\infty &= A'S_\infty A + W_t - \bar{\nu}A'S_\infty B(W_t + B'S_\infty B)^{-1}BS_\infty A \\
\hat{P}_\infty &= A\hat{P}_\infty A' + Q - \bar{\gamma}A\hat{P}_\infty C'(C\hat{P}_\infty C' + R)^{-1}C\hat{P}_\infty A'
\end{aligned}
$$

($\bar{\nu} \rightarrow 1$)



LQG Controller Design: UDP case

Scalar system, i.e. x2R

$$\begin{aligned} x_{t+1} &= x_t + \nu_t u_t \\ y_t &= x_t + v_t, \qquad \mathrm{cov}(v_t) = \gamma_t 1 + (1-\gamma_t)\sigma^2 I \end{aligned}$$

$$J_N = \mathbb{E}[\sum_{t=0}^{N} |x_t|^2]$$

$$\mathcal{I}_t = \{y_t, \gamma_t, ...\}$$

t=N

$$V_N(x_N) = \mathbb{E}[x_N^2]$$

t=N-1

$$\begin{aligned} V_{N-1}(x_{N-1}) &= \min_{u_{N-1}} \mathbb{E}[x_{N-1}^2 + V_N(x_N) \mid \mathcal{I}_{N-1}] \\ &= \min_{u_{N-1}} \mathbb{E}[x_{N-1}^2 + x_N^2 \mid \mathcal{I}_{N-1}] \\ &= \min_{u_{N-1}} \left(\mathbb{E}[x_{N-1}^2 + x_{N-1}^2 \mid \mathcal{I}_{N-1}] + \bar{\nu} u_{N-1}^2 + 2\bar{\nu} u_{N-1} \hat{x}_{N-1|N-1}\right) \\ &= 2\,\mathbb{E}[x_{N-1}^2 \mid \mathcal{I}_{N-1}] - \bar{\nu}\hat{x}_{N-1|N-1}^2, \qquad u_{N-1}^* = -\hat{x}_{N-1|N-1} \\ &= (2-\bar{\nu})\,\mathbb{E}[x_{N-1}^2 \mid \mathcal{I}_{N-1}] + \bar{\nu} P_{N-1|N-1} \end{aligned}$$

LQG Controller Design: UDP case

t=N-2

$$\begin{aligned} V_{N-2}(x_{N-2}) &= \min_{u_{N-2}} \mathbb{E}[x_{N-2}^2 + V_{N-1}(x_{N-1}) \mid \mathcal{I}_{N-2}] \\ &= \mathbb{E}[(3-\bar{\nu})x_{N-2}^2 \mid \mathcal{G}_{N-2}] + \bar{\gamma} + \bar{\nu}(2-\bar{\gamma})P_{N-2|N-2} + \\ &\quad + \min_{u_{N-2}} \Big((2\bar{\nu} - \bar{\nu}^3 - \bar{\nu}^2\bar{\gamma} + \bar{\nu}^3\bar{\gamma})u_{N-2}^2 + 2\bar{\nu}(2-\bar{\nu})\hat{x}_{N-2|N-2} u_{N-2} + \\ &\quad + \bar{\nu}\bar{\gamma}\frac{1}{P_{N-2|N-2} + \bar{\nu}(1-\bar{\nu})u_{N-2}^2 + 1}\Big) \end{aligned}$$

$$u_t^* = k_t(y_t, ..., y_0, \gamma_t, ..., \gamma_0, P_0, x_0)$$

NONLINEAR FUNCTION OF INFORMATION SET *It*

UDP controller: Estimator design
Special case: C invertible, R=0

Without loss of generality I can assume C=I

prediction

$$P_{t+1|t} = AP_{t|t}A' + Q + \bar{\nu}(1-\bar{\nu})Bu_tu_t'B$$

correction

$$\begin{aligned} P_{t+1|t+1} &= P_{t+1|t} - \gamma_{t+1}P_{t+1|t}C'(CP_{t+1|t}C'+R)^{-1}CP_{t+1|t} \\ &= (1-\gamma_t)P_{t+1|t} \end{aligned}$$

$$\begin{aligned} P_{t+1|t+1} &= (1-\gamma_t)(AP_{t|t}A' + Q + \bar{\nu}(1-\bar{\nu})Bu_tu_t'B) \\ K_{t+1} &= I \\ \hat{x}_{t+1|t+1} &= (1-\gamma_{t+1})\hat{x}_{t+1|t} + \gamma_{t+1}y_{t+1} \end{aligned}$$

UDP special case:
C invertible, R=0

It is possible to show that:

$$V_k(x_t) = \mathbb{E}[x_t'S_tx_t \mid \mathcal{I}_t] + \text{trace}(T_tP_{t|t}) + \text{trace}(D_tQ)$$

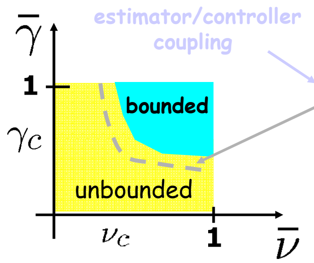
$$\begin{aligned} u_t^* &= -\left(U_t + B'\left(S_{t+1} + (1-\bar{\nu})(1-\bar{\gamma})T_{t+1}\right)B\right)^{-1}B'S_{t+1}A\;\hat{x}_{t|t} \\ &= L_t\,\hat{x}_{t|t} \end{aligned}$$

$$S_N = W_N, \quad T_N = 0, \quad D_N = 0$$

$$\begin{aligned} S_t &= A'S_{t+1}A + W_t - \bar{\nu}A'S_{t+1}B(U_t + B'(S_{t+1} + (1-\bar{\nu})(1-\bar{\gamma})T_{t+1})B)^{-1}B'S_{t+1}A \\ T_t &= (1-\bar{\gamma})A'T_{t+1}A + A'S_{t+1}A + W_t - S_t \\ D_t &= D_{t+1} + (1-\bar{\gamma})T_{t+1} + S_{t+1} \end{aligned}$$

UDP Infinite Horizon: C invertible, R=0

It is possible to show that:



Necessary condition for boundedness:

$$\bar{\nu}\bar{\gamma} > 1 - \frac{1}{|\lambda_{max}(A)|^2}$$

Sufficient only if B invertible

Conclusions

- Closed the loop around sensor networks
- General framework applies to networked control system
- Solved the optimal control problem for full state feedback linear control problems
- Bounds on the cost function
- Transition from state boundedness to instability appears
- Critical network values for this transition

Thank you !!!

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Related publications:

- Kalman Filtering with Intermittent Observations
  - *IEEE TAC September 2004*
- Time Varying Optimal Control with Packet Losses
  - *IEEE CDC 2004*
- Optimal Control with Unreliable Communication: the TCP Case
  - *ACC 2005*
- LQG Control with Missing Observation and Control Packets
  - *IFAC 2005*