# Thermal Modeling for Buildings

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# Overview

- Motivation
- Problem Statement
- Related Work
- Heat Transfer Basics
- Proposed Approach
- Model
  - SimMechanics
  - Matlab PDE
- Control Effort
- Summary

# Motivation

- Buildings account for approximately 40% of world energy use -> 21% of greenhouse gas emissions
- In the U.S., buildings contribute 1 billion metric tons of greenhouse gas emissions.
- On an annual basis, buildings Consume
  - 39% of total U.S. Energy
  - 68% of U.S Electricity
  - 38% of U.S. carbon dioxide (primary greenhouse gas associated with climate change)
  - 25% of the nitrogen oxides found in the air
- The only energy end-use sector showing growth in energy intensity

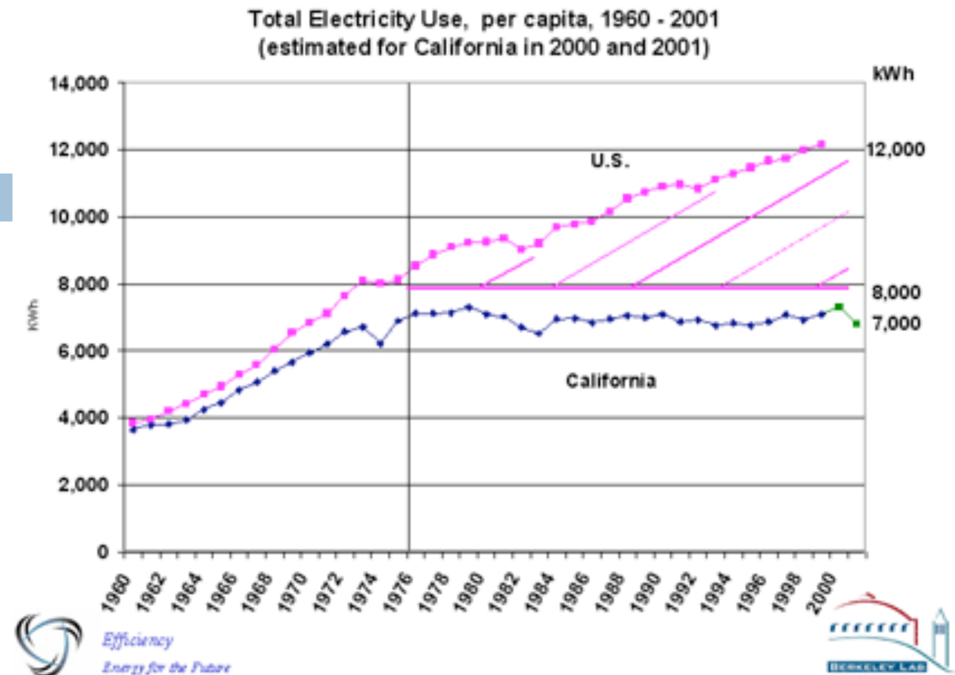
Source: National Institute of Building Sciences 2009, EE249 Lecture, Fall 2008 Report

# Motivation

- According to the National Institute of Building Sciences, an Intelligent Buildings must seek to:
  - Reduce heating, cooling and lighting loads through climate-responsive design and conservation practices
  - Employ renewable energy sources (daylighting, passive solar heating, photovoltaics, geothermal, and groundwater cooling)
  - Specify efficient HVAC and lighting systems that consider part-load conditions and utility interface requirements
  - Optimize building performance by employing energy modeling programs and optimize system control strategies
  - Monitor project performance
- Energy codes provide minimum building requirements that are cost-effective in saving energy

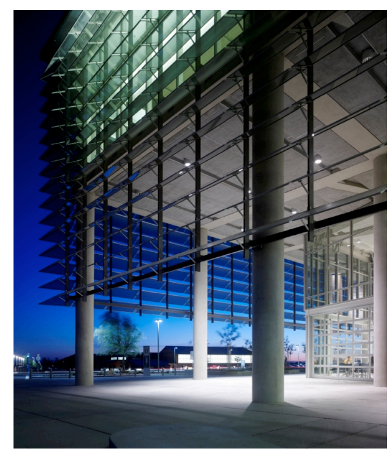
# Motivation

- The UC Merced campus: full scale test-bed for advanced energy management in campus buildings and systems
- Test-bed facility involves design, implementation and evaluation of a sensor network prototype
- Sensors include:
  - Light
  - Temperature
  - Humidity
  - Light intensity
  - Mobility patterns of humans inside building
- New measurements can help improve and design control Energy Management and Control Systems (EMCS)




Total Energy Use, per Capita 1960-2001 (Estimated for CA for 2000 and 2001

National Institute of Building Sciences 2009



UC Merced Campus

# Problem Statement

- In order to investigate how new technologies can help improve energy usage at the UC-Merced buildings, an scalable thermal model of the building needs to be created, verified and validated.
- Scalability is important when analyzing control systems for large buildings. Relevant to hybrid control system models
- Objectives
  - Create a scalable model (in Simulink) of building thermodynamics
  - Model UC Merced building
  - Validate model using reference data of UC Merced

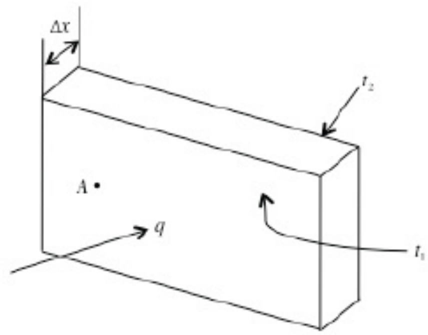
# Related Work

- Maasoumy, Holcomb [Fall 2008]. High Performance Building. Simulink modeling and control. More emphasis on control systems
- Schijndel [2003]. FemLab (Physical system simulator) and Simulink are evaluated as solvers for HVAC problems based on PDEs. Proposes the use in combination of both system, but doesn't address scalability.
- Mendes, et al [2001]. Mathematical model applied to both building thermal analysis and control system design using MatLab and Simulink. Approach incorporates multilayer walls, but does not address scalability.
- Felgner, et al [2002]. Modelica is used. Modelica has libraries for building thermal characteristics. Paper makes a good argument about the power of Modelica. However, feedback from Control Engineering at the ME department reveals that it is not an easy tool to use.
- Others: Address in detail HVAC systems, but do not address plant modeling or control systems.

# Heat Transfer Basics

- Conduction

$$q = kA\frac{t_1 - t_2}{\delta x}$$




- Convection

$$\frac{q}{A} = h(t_s - t_f)$$

- Radiation

$$E = \epsilon\sigma T^4$$

**Nomenclature**

| | |
|---|---|
| $C_p$ | Specific Heat |
| $\rho$ | Density |
| $t$ | Temperature, K |
| $\tau$ | Time, s |
| $m$ | Mass, kg |
| $q$ | Heat flow |
| $h$ | Heat transfer coefficient of the material |
| $k$ | Conduction coefficient |
| $L$ | Length of conductor, m |
| $A$ | Area, $m^2$ |

- Heat Storage
  - Heat capacity describes how much energy is required to increase the temperature by a specified amount. Proportional to the mass of the object

$$mC_p\frac{dt}{d\tau} = q$$

# Heat Transfer Basics: Thermal Circuits

- Useful concept in the representation of thermal transfer
- Thermal circuit is a representation of the resistance to heat flow as though it were a resistor
- Heat storage elements can be represented as capacitors
- Temperature can be represented as potential

- Network nodal analysis, the following must be satisfied at each node

$$\sum_j \frac{t_j - t_i}{R_{ij}} + q_i = 0$$

- Where $q_i$ represents heat added to the node by means other than surface convection. If internal heat is present, the $q_i$'s are known

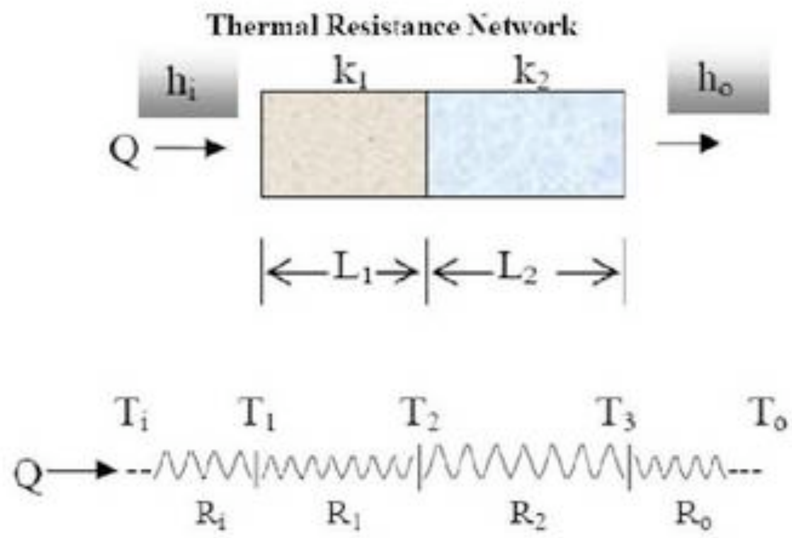
# Heat Transfer Basics: Thermal Circuits

- For conduction the resistance is:

$$R = \frac{\delta x_{ij}}{k A_{k_{ij}}}$$

- For convection it is:

$$R = \frac{1}{h_{ij} A_{c_{ij}}}$$



# Proposed Approach

- Simscape from MathWorks
  - Extends the Simulink with tools for modeling systems spanning mechanical, electrical, hydraulic and other physical domains and physical networks
  - For our approach, use the electrical physical components to simulate a room and investigate its scalability
- CHALLENGES
  - No UC Merced model data for validation
  - Mitigation: model using other tool (ComSol, Modelica or PDE Matlab)
  - Other tools have higher learning curve. Verified the Model against Matlab PDEs

# Model: Analytical

- Since building is a complex system, a complete theoretical approach is impractical.
- Assumptions:
  - Air is the zone is fully mixed. Temperature distribution is uniform and the dynamics can be expressed in a lump capacity model
  - Effect of each wall is the same
  - Ground and roof have no effect on the zone temperature
  - The density of the air is assumed to be constant and is not influenced by changing the temperature and humidity ratio of the zone

# Model: Description

- State variables:
  - Zone temperature and wall temperatures
- People, lights and extreme weather conditions are uncontrolled inputs.
- Input variables:
  - Air flow rate for each zone

# Model: Analytical Model

$$\frac{dt_i}{d\tau} = \frac{1}{C_w}\left[q_{rad} + \frac{t_o - t_i}{Ri_{w_i}} + \frac{t_a - t_i}{Ro_{w_i}}\right] \tag{1}$$

Where

$$C_w = LA\rho_w C_{pw} \tag{2}$$

$$Ri_{w_i} = \frac{Rw_i}{2} + R_{in} \tag{3}$$

$$Rw_i = \frac{L}{KA} \tag{4}$$

$$R_{in} = \frac{1}{h_{in}A} \tag{5}$$

$$Ro_{w_i} = \frac{Rw_i}{2} + R_{out} \tag{6}$$

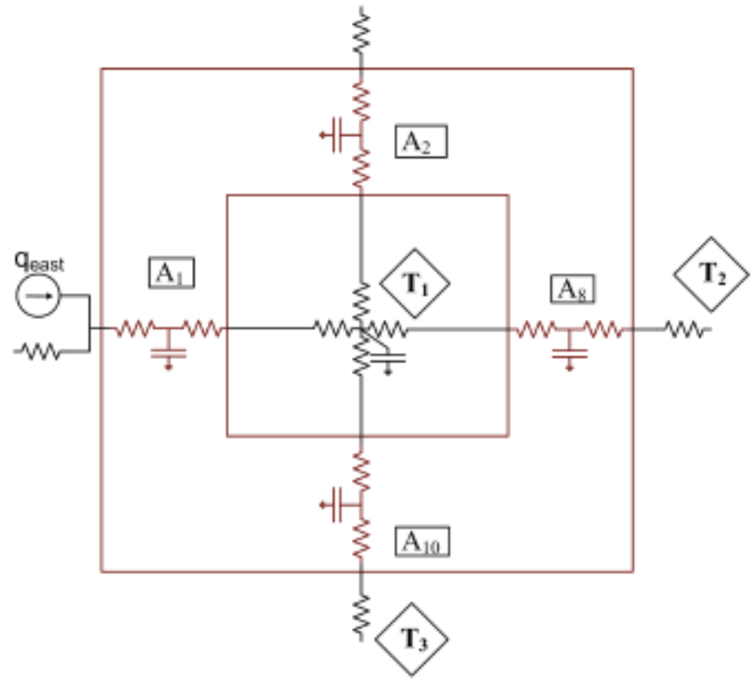
$$R_{out} = \frac{1}{h_{out}A} \tag{7}$$

$$\frac{dt_a}{d\tau} = \frac{1}{C_a}\left[\sum_{i=1}^{n}\frac{t_i - t_a}{Ri_{w_i}} + q_{in} + q_{others}\right] \tag{8}$$

$$q_{in} = \dot{m}C_{pa}\left(t_{input} - t_a\right) \tag{9}$$

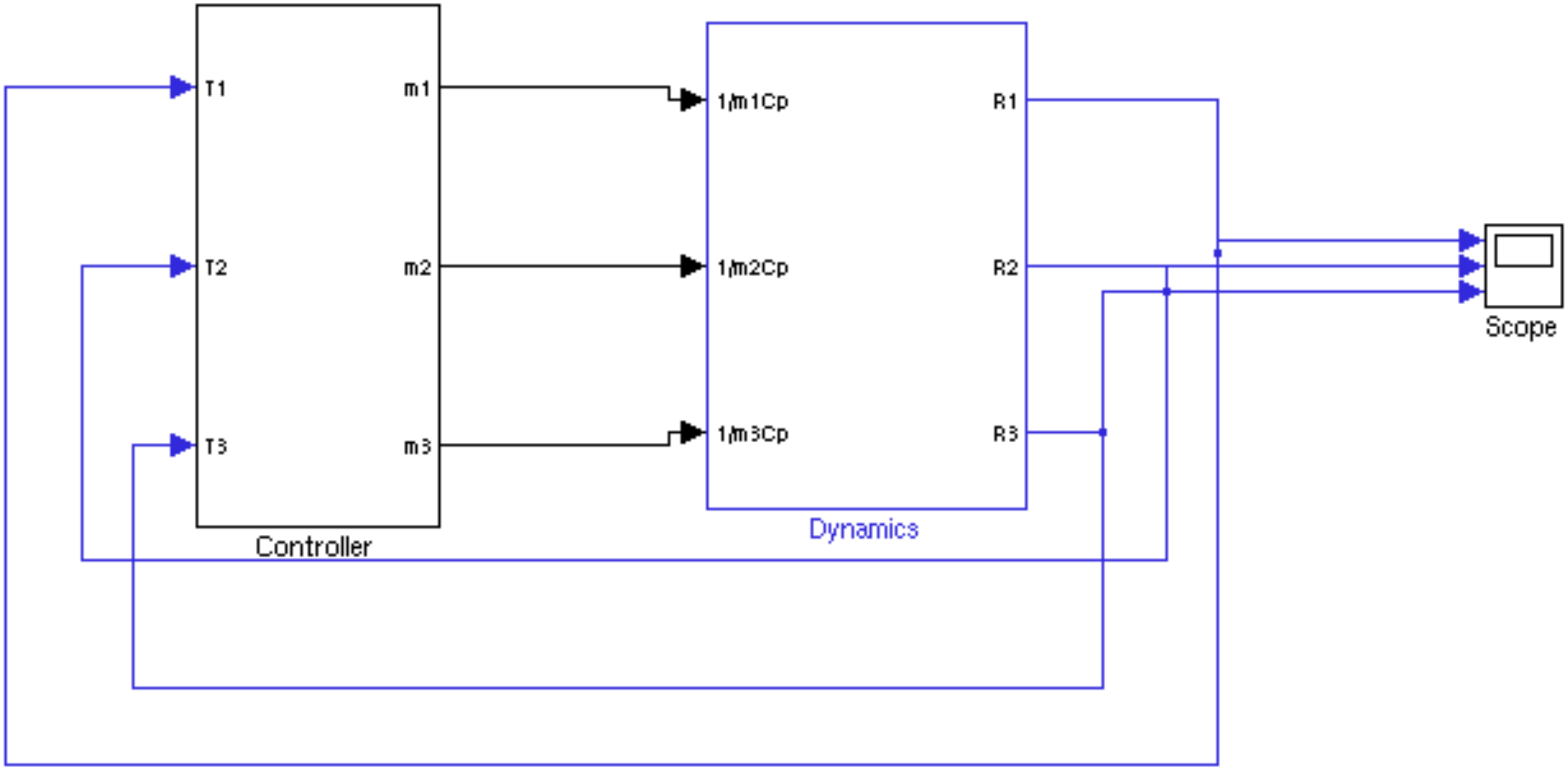
- Other heat gains can be set to zero.

$$D(\tau) = \sum_{j=1}^{m}\frac{t_{eq(\tau)} - t_A(\tau)}{R_j} + q_p + q_l + q_o$$



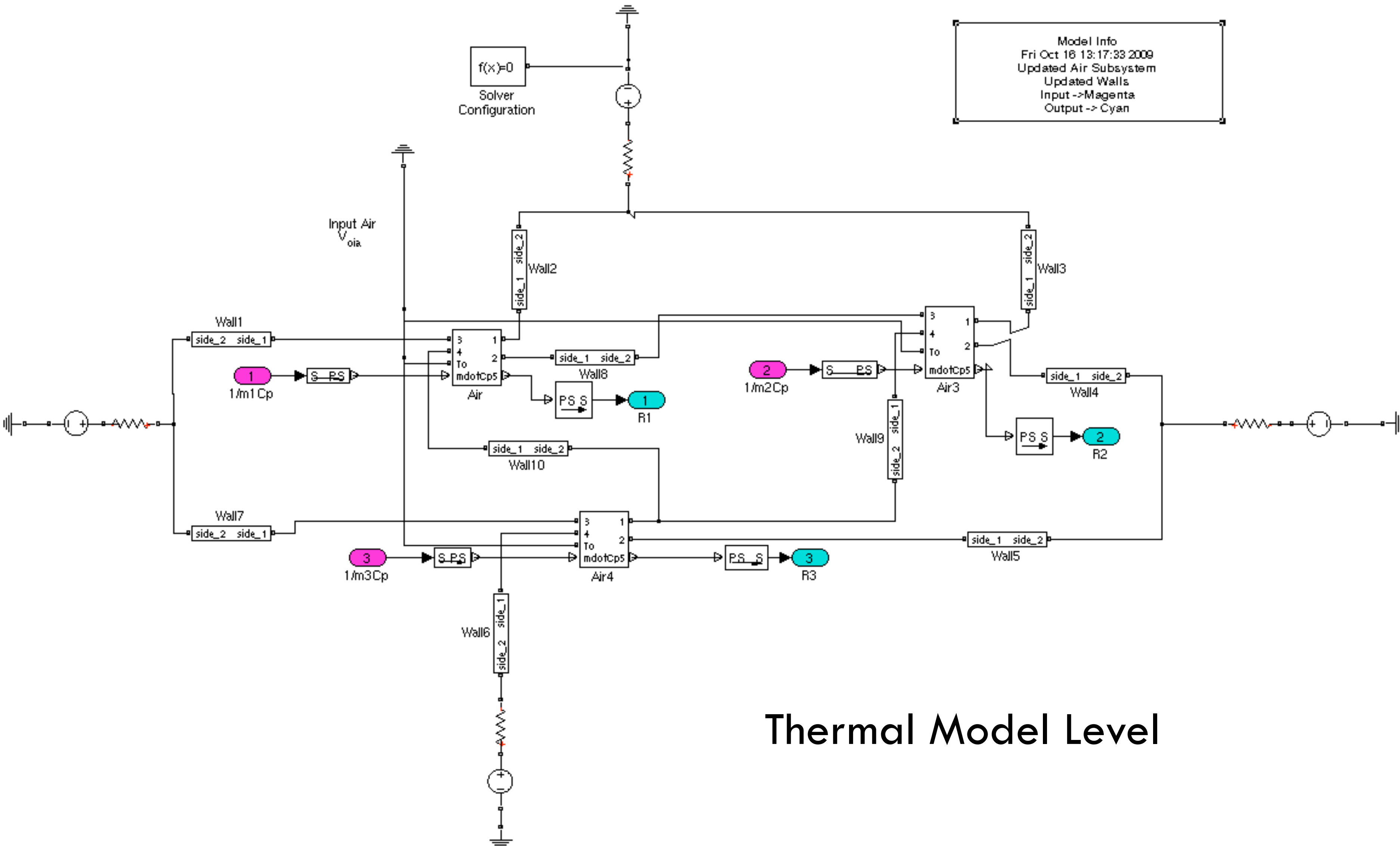

Fall 2008 Room Model

# Model: Simulink (1 and 3 Rooms)



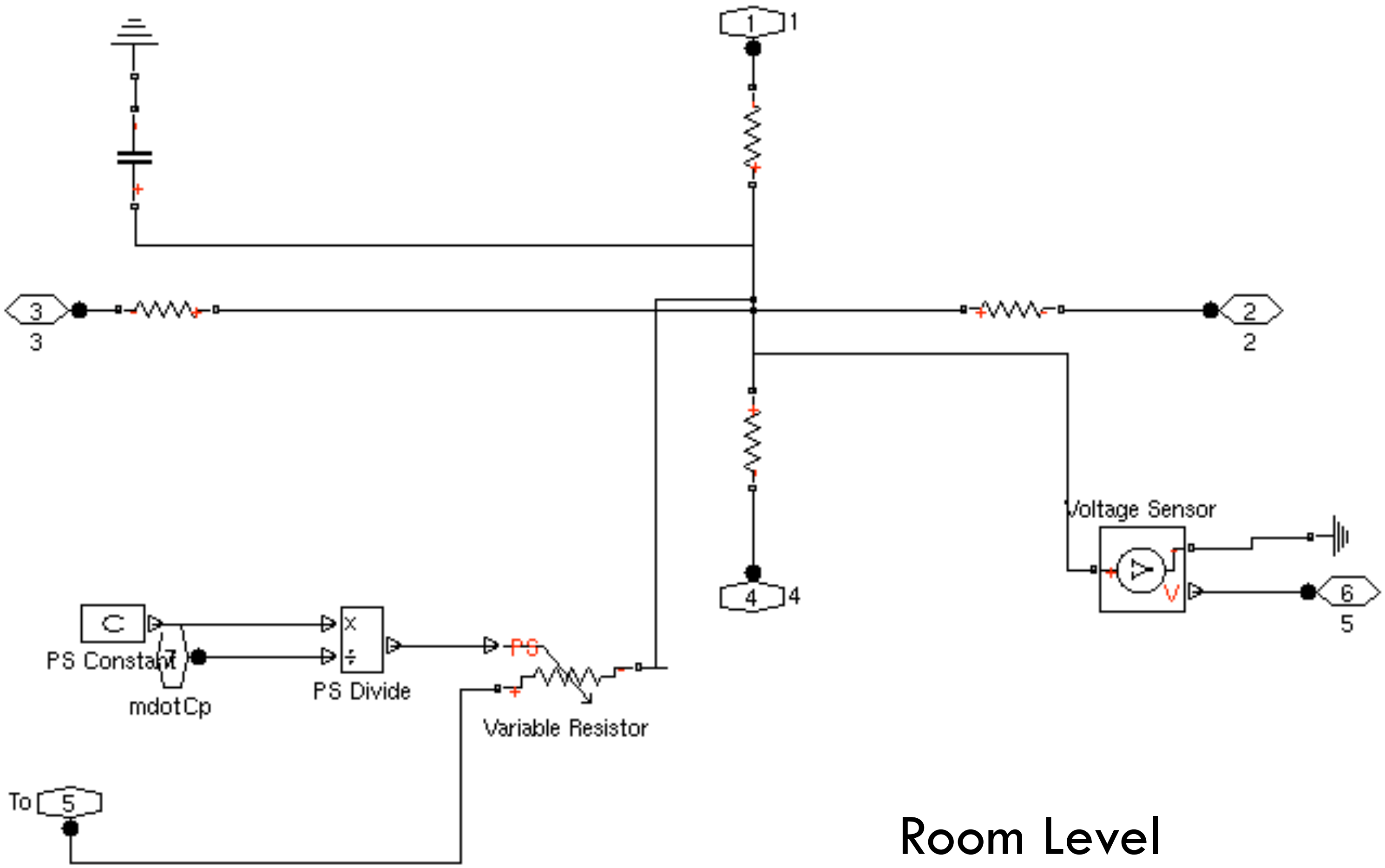

Top Level on Simulink

# Model: Simulink (1 and 3 Rooms)



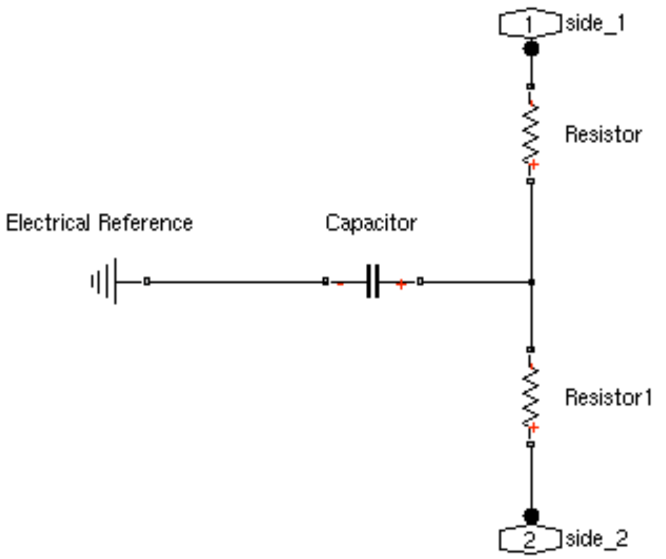

Thermal Model Level

# Model: Simulink (1 and 3 Rooms)



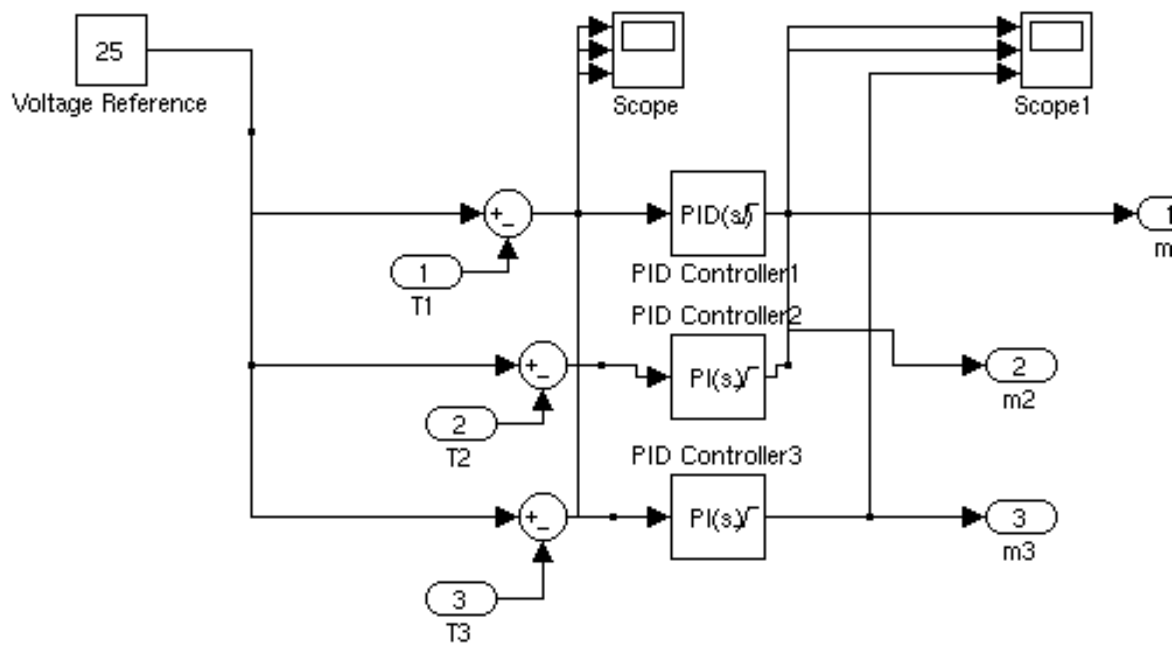

Room Level

# Model: Simulink (1 and 3 Rooms)

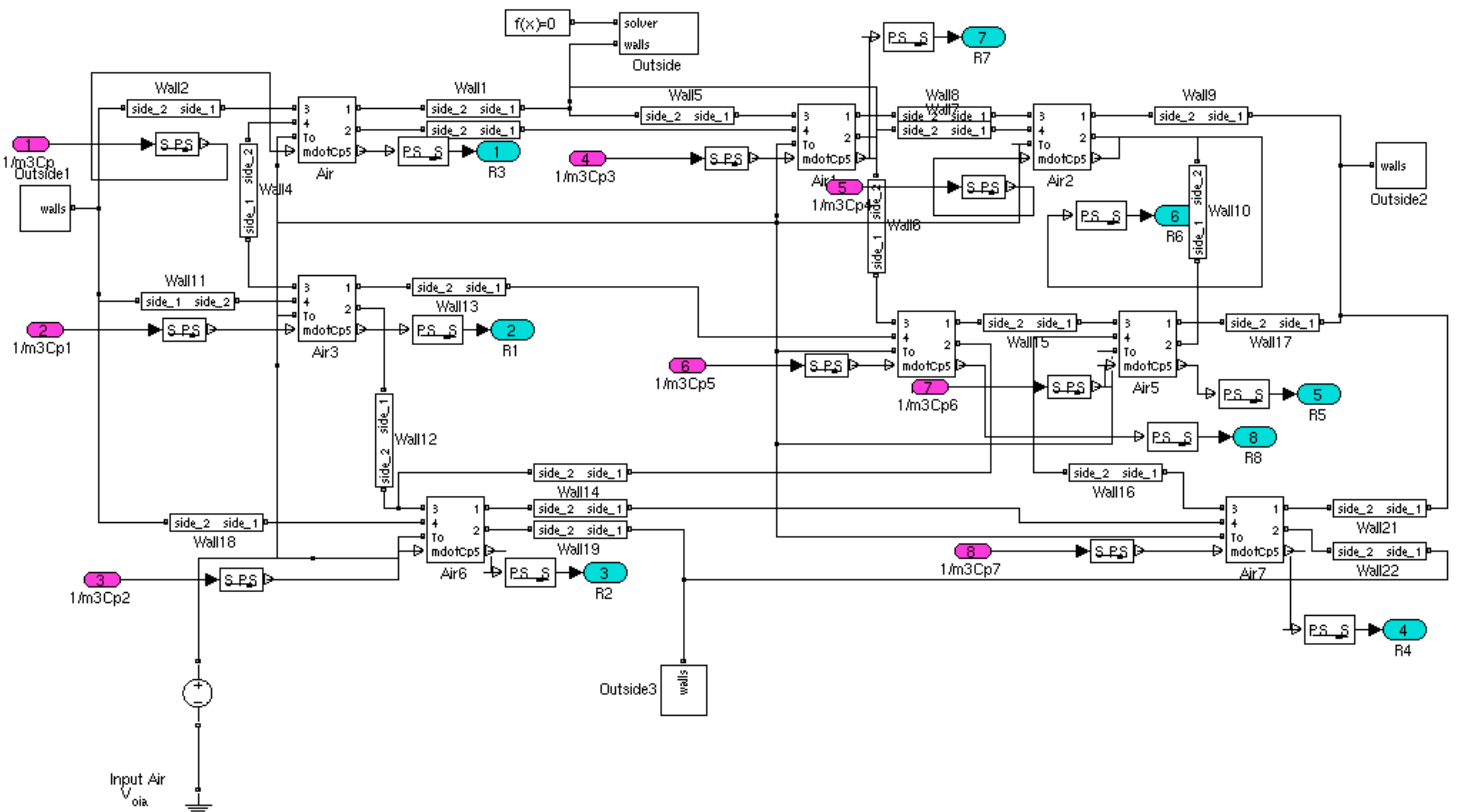



Wall Level

Control Block



# Model: Simulink 8 rooms



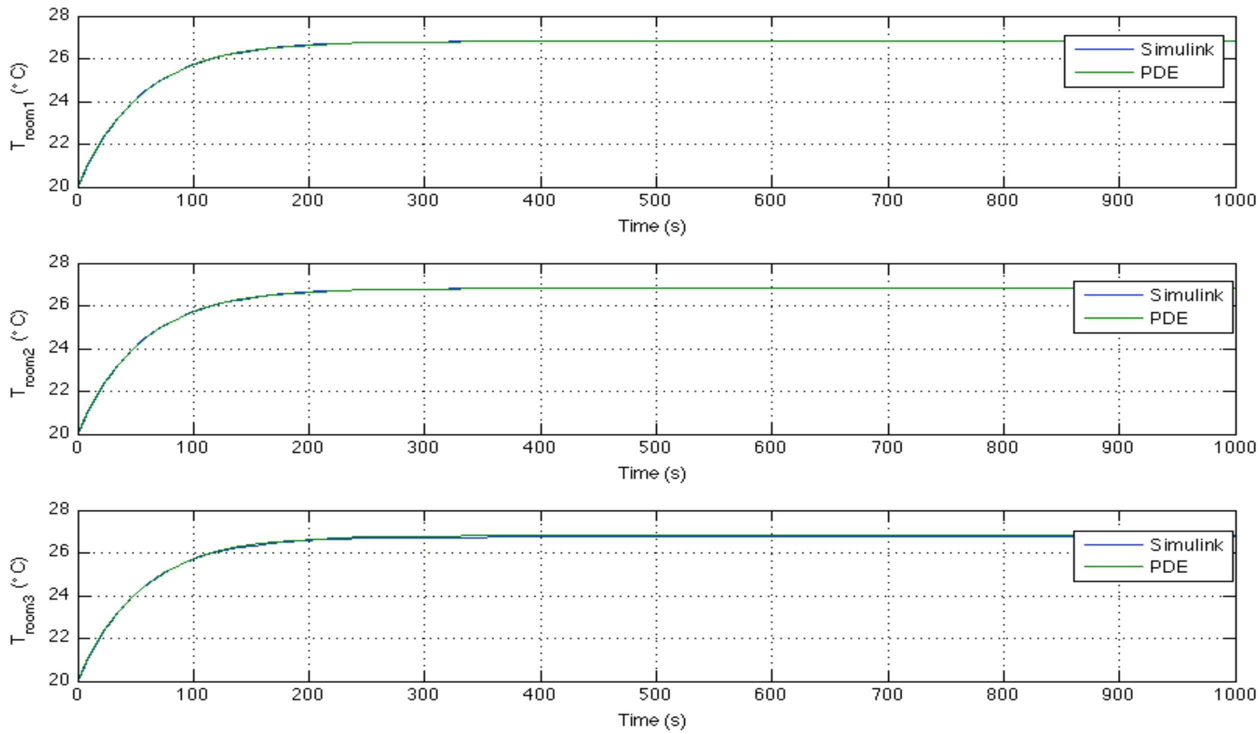
# Model: PDEs MatLab

```
global v_outside
global v_input
global Rw
global R_out
global Ra
global Cw
global Ca
global Cp_a
global input

h_in = 5; % W/(m^2 K)
h_out = 20; % W/(m^2 K)
k = .6;
rho =1500; %density
rho_air = 1;
Cp_w = 1000;% J/(kg K) specific heat capacitance wall
Cp_a = 1000;% J/(kg K) specific heat capacitance ...Air

% initial voltage for capacitors
vo_c = 20; % Volts
vo_air = 20; % Volts
v_outside = 15; % Volts
v_input = 30; %volts

% Wall Dimensions

L = 0.2;% Length m (thickness)
W = 4;% width m
H = 4;% Height m
A = W*H; % Area m^2

%% Inputs to Room Subsystem

Ra = 1/(h_in*A); % Resistance to Air Subsystem Rconvection_in
R_out = 1/(h_out*A); % Rconvection_out
Rw = L/(k*A);%Resistance of wall
Cw = L*A*rho*Cp_w;%Capacitance of wall = mCa
Ca = W^2*H*rho_air*Cp_a;

%
input = [0;1;0];
% state T, Tw1, Tw2,Tw3,Tw4
xo = [vo_air;vo_air; vo_air; vo_c;vo_c;vo_c;vo_c;vo_c;vo_c;vo_c;vo_c;vo_
tspan = [0 1000];
options = odeset('RelTol', 1e-3, 'AbsTol', 1e-3);
[t,x] = ode45(@deriv2, tspan, xo, options);
```

```
function x_dot = deriv2(dt,x)

global v_outside
global v_input
global Rw
global R_out
global Ra
global Cw
global Ca
global Cp_a
global input

To = v_outside;
Ta_1 = x(1);
Ta_2 = x(2);
Ta_3 = x(3);
cnt = 3;
T_room1 = [x(cnt+1);x(cnt+2);x(cnt+8);x(cnt+10)];
T_room2 = [x(cnt+3);x(cnt+4);x(cnt+8);x(cnt+9)];
T_room3 = [x(cnt+5);x(cnt+6);x(cnt+7);x(cnt+9);x(cnt+10)];
Riw1 = Rw/2 + Ra;
Row1 = Rw/2 + R_out;
qrad = 0;


% Differential equation of the walls
% room1
dTw(1,1) = (1/Cw)*(qrad + (To - T_room1(1))/Riw1 + (Ta_1 - T_room1(1))/Row1);
dTw(2,1) = (1/Cw)*(qrad + (To - T_room1(2))/Riw1 + (Ta_1 - T_room1(2))/Row1);
dTw(8,1) = (1/Cw)*(qrad + (Ta_2 - T_room1(3))/Riw1 + (Ta_1 - T_room1(3))/Row1);
dTw(10,1) = (1/Cw)*(qrad + (Ta_3 - T_room1(4))/Riw1 + (Ta_1 - T_room1(4))/Row1);

% room2
dTw(3,1) = (1/Cw)*(qrad + (To - T_room2(1))/Riw1 + (Ta_2 - T_room2(1))/Row1);
dTw(4,1) = (1/Cw)*(qrad + (To - T_room2(2))/Riw1 + (Ta_2 - T_room2(2))/Row1);
dTw(9,1) = (1/Cw)*(qrad + (Ta_3 - T_room2(4))/Riw1 + (Ta_2 - T_room2(4))/Row1);

% room3
dTw(5,1) = (1/Cw)*(qrad + (To - T_room3(1))/Riw1 + (Ta_3 - T_room3(1))/Row1);
dTw(6,1) = (1/Cw)*(qrad + (To - T_room3(2))/Riw1 + (Ta_3 - T_room3(2))/Row1);
dTw(7,1) = (1/Cw)*(qrad + (To - T_room3(3))/Riw1 + (Ta_3 - T_room3(3))/Row1);
```

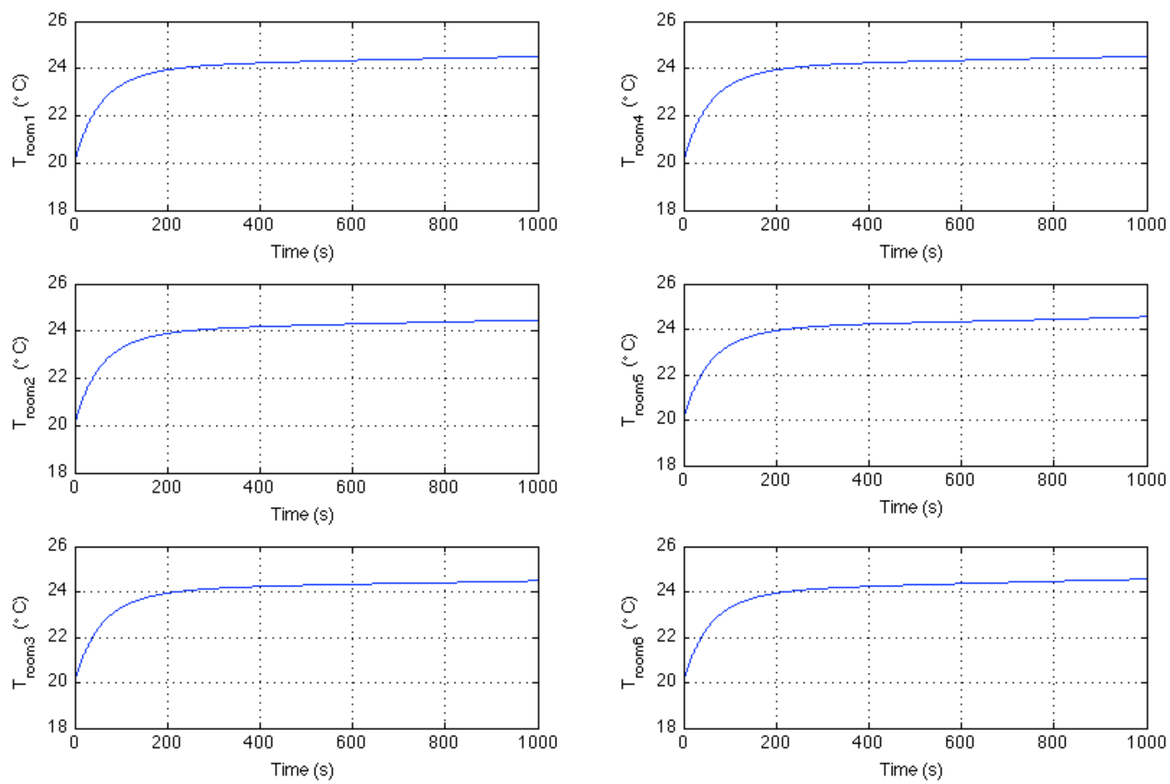
# Model: Results PDE vs. Simulink (constant input)

- Error is in the order of e-9, probably due to numerical integration errors.



# Model Results: Scalability-PID 8 room model

- Set point is 25 degrees C. Compute time increases, but model is easily scalable on Simulink. If wall properties are similar, Matlab model is easily scalable as well



# Control Effort

- PID Model for room uses the desired temperature errors as input. Very easy to implement on Simulink.
- Other advance control techniques (LQR, CLQR, MPC) require state space representation. Scalability can be a problem and state-space representation is not easily implementable.
- No direct state-space extension from circuit analysis

# Control Effort

- For the 3-room model system we can define the state space as:

- Where x is the state and u is the input vector. This non-linear system can be linearized about a nominal trajectory ($x_n$,$u_n$) for implementation of LQR or CLQR. The constraint is the input since the amount of airflow should be r

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + diag(\mathbf{x}^T\mathbf{R_1})\mathbf{R_2}\mathbf{u} + \mathbf{B}\mathbf{u}$$

# Control Effort

- The matrices A, $R_1$, $R_2$ and B are defined by the dynamics of the system. $R_1$ and $R_2$ are necessary to represent the non-linearity in the model.
- Linearization is straight forward. Selection of nominal trajectory is not.
- Future work should focus on providing the correct system dynamics to the controller.

# Summary

- A scalable model of a thermal system was presented. The model was verified using another modeling tool. Behavior of the systems to same inputs produce same outputs. Errors are only due to numerical integration
- Validation and model tuning can be done once UC Merced data is available
- Advanced control techniques require a different definition of model (state-space). A simple representation was shown, which can be linearized about a nominal trajectory
- Future work should include the proper representation of the non-linear model for advanced control techniques