Scheduling Algorithms for Multiprogramming in a Hard-Real-Time Environment

Presenters: Forrest Iandola and Ilge Akkaya
(thanks to Marco Di Natale and Tarek Abdelzaher)

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Scheduling Algorithms for Multiprogramming in a Hard-Real-Time Environment

(The Liu-Layland Bound)

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Outline

Goals
 Synthesize Marco Di Nitale’s discussion of the Liu-Layland bound
 Review the key takeaways on this topic

Roadmap
 State the Liu-Layland bound
 Do some short examples, build up intuition
 …and move on!
Is a task set schedulable?

Schedulability analysis strategies

- Utilization bounds (easy but pessimistic)
  - Lower utilization -> easier to meet deadlines
  - Higher utilization -> harder to meet deadlines
- Exact analysis (optimal, but NP-hard)
- Heuristics
Is a task set schedulable?

Schedulability analysis strategies

- Utilization bounds (easy but pessimistic)
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- Heuristics
The Liu-Layland Bound

- Utilization bound for $n$ periodic tasks:
  
  $$U = n \left( 2^{\frac{1}{n}} - 1 \right)$$

- For $n=2$ tasks, $U = 0.83$
- As $n$ goes to infinity, $U = 0.69$

- Assumption: fixed-priority scheduling
The bound is NOT “if and only if.”

- If system utilization is within the bound, it is \textit{guaranteed} to be schedulable.
- If system utilization exceeds the bound, it \textit{may} be schedulable.

\[
\text{Utilization} = \sum_{i} \frac{C_i}{P_i}
\]
Key insight: Blocking time

- 2 tasks, both arrive at the same time
- Notice that the lower-priority task waits for a long time
Conclusion

- Liu-Layland bound:
  \[ U = n \left( 2^{\frac{1}{n}} - 1 \right) \]

- Assuming worst-case arrival and blocking times leads to less-than-100% utilization bound

- If a system exceeds the bound, it may still be schedulable