A Robot delivery service, with moving obstacles

\[ \phi = \text{destination for robot} \]

At any time step:
- Robot can move Left, Right, Up, Down, Stay Put
- Environment can move one obstacle Up or Down or Stay Put
  - But only 5 times total
- Can model Robot and Env as FSMs
  - Robot state = its position,
  - Env state = positions of obstacles and counts
A Robot delivery service, with moving obstacles

\[ \phi = \text{robot delivers item to destination} \]
Goal to be achieved can be stated in temporal logic

\[ F \phi \]

How can we find a path for the robot from starting point to the destination?
\[ \rightarrow \text{This is an example of a “reachability problem”} \]

Solving the Reachability Problem

- Construct FSMs for Robot and Environment
- Compose FSMs to form a new FSM
- Check whether the goal state is reachable from the start state
Open vs. Closed Systems

A closed system is one with no inputs and no outputs

Typically, the system we analyze is the composition of the system with its environment

- this composed system is closed!
- In general, could be non-deterministic (why?)

Recall: a ND FSM is a 5-tuple

(States, Inputs, Outputs, possibleUpdates, initialStates)

For a closed system, Inputs = Outputs = ∅

Denote:

- States – Q, possibleUpdates – δ, initialStates – Q₀
Reachability Analysis for FSMs

The reachability problem:

Given an FSM $M = (Q, \delta, Q_0)$, and a state $s$, is $s$ reachable from some $q_0 \in Q_0$ by following $\delta$?

Note: Although we write $Q$ as part of $M$, typically we don’t have the set of states $Q$
- it’s too large!!!
Reachability Analysis for FSMs

The reachability problem:

Given an FSM \( M = (Q, \delta, Q_0) \), and a state \( s \), is \( s \) reachable from some \( q_0 \in Q_0 \) by following \( \delta \)?

How can we express the property “\( s \) is reachable from a start state” in Temporal Logic?

Outline of Approach

- Generate the state graph by repeated application of \( \delta \)
- If the goal state reached, stop & report success. Else, continue until all states are seen.
The Reachability Algorithm

Input: Description of M: \((Q_0, \delta), s\)
Output: Is \(s\) reachable from \(Q_0\)?

Init: \(S := S_{\text{new}} := Q_0\);
while \((S_{\text{new}} \neq \emptyset)\) {
    if \((s \in S_{\text{new}})\)
        return YES;
    \(S' := \{ q \mid \exists p \in S \text{ s.t. } q \in \delta(p) \} \cup S\)
    \(S_{\text{new}} := S' \setminus S\);
}

Suppose we have a Robot that must pick up multiple things, in any order

\[ \phi_i = \text{robot picks up item } i, \quad 1 \leq i \leq n \]

How would you state this goal in temporal logic?
Suppose we have a Robot that must pick up multiple things, in any order

\[ \phi_i = \text{robot picks up item } i, \quad 1 \leq i \leq n \]

Goal to be achieved is:

\[ F \phi_1 \wedge F \phi_2 \wedge \ldots \wedge F \phi_n \]

How can we find a strategy to achieve this goal?
Suppose we have a Robot that must pick up multiple things, in any order

\[ \phi_i = \text{robot picks up item } i, \quad 1 \leq i \leq n \]

Goal to be achieved is:

\[ F (\phi_1 \land F (\phi_2 \land \ldots \land F (\phi_n))) \]

How can we find a strategy to achieve this goal?

→ Do repeated reachability, first from \(Q_0\) to reach \(\phi_1\), then from \(\phi_1\) to reach \(\phi_2\), then \(\phi_2\) to reach \(\phi_3\), ...

Student question: Suppose we have a Robot that must pick up multiple things, in a specified order

\[ \phi_i = \text{robot picks up item } i, \quad 1 \leq i \leq n \]

Goal to be achieved is:

\[ F (\phi_1 \land F (\phi_2 \land \ldots \land F (\phi_n))) \]
Exercise

Can we do reachability analysis backwards?

I.e.: start with $s$ instead of $Q_0$, and then follow the update function $\delta$ backwards…