



# Introduction to Embedded Systems

Sanjit A. Seshia

UC Berkeley  
EECS 149/249A  
Fall 2015

© 2008-2015: E. A. Lee, A. L. Sangiovanni-Vincentelli, S. A. Seshia. All rights reserved.

**Chapter 5: Composition of State Machines**

## Composition of State Machines

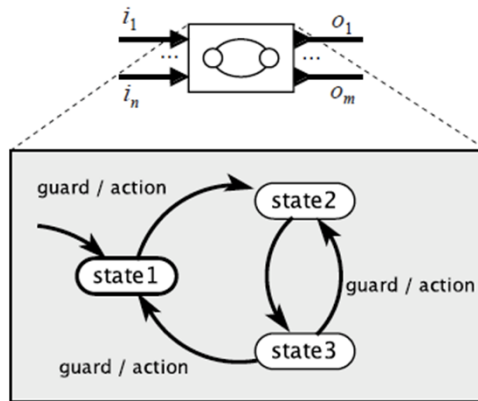
How do we construct complex state machines out of simpler “building blocks”?

Two kinds of composition:

1. **Spatial**: how do the components communicate between each other?
2. **Temporal**: when do the components execute, relative to each other?

## Actor Model for State Machines

Expose inputs and outputs, enabling composition:



EECS 149/249A, UC Berkeley: 3

## Spatial Composition of State Machines

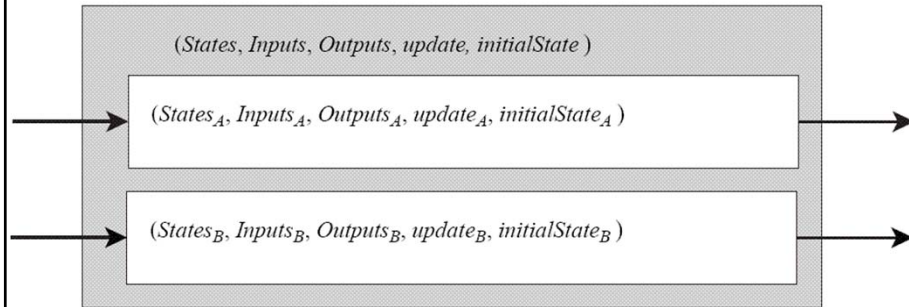
Side-by-side composition

Cascade composition

Feedback composition

EECS 149/249A, UC Berkeley: 4

## Side-by-Side Composition



A key question: When do these machines react?

EECS 149/249A, UC Berkeley: 5

## Temporal Composition of State Machines

Sequential vs. Parallel

Asynchronous vs. Synchronous

EECS 149/249A, UC Berkeley: 6

## Example of Sequential Composition



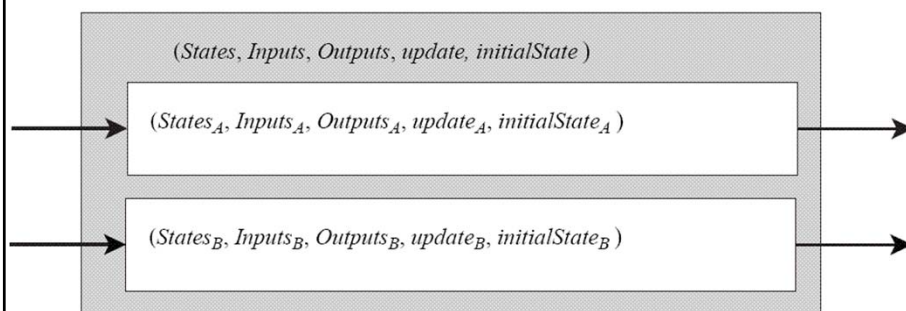
<https://www.youtube.com/watch?v=iD3QgGpzzIM>



[Tomlin et al.]

EECS 149/249A, UC Berkeley: 7

## Side-by-Side, Parallel Composition



When do these machines react?

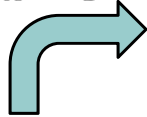
Two possibilities:

- Together, in lock step (synchronous, parallel composition)
- Independently (asynchronous, parallel composition)

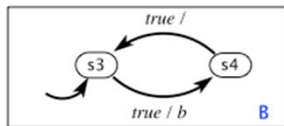
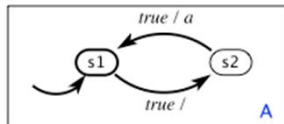
EECS 149/249A, UC Berkeley: 8

## Synchronous Composition

$$S_C \subseteq S_A \times S_B$$



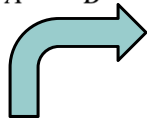
outputs:  $a, b$  (pure)



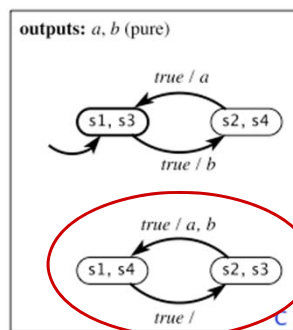
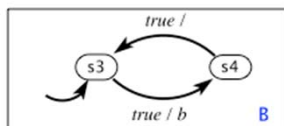
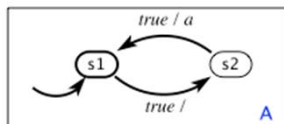
EECS 149/249A, UC Berkeley: 9

## Synchronous Composition

$$S_C \subseteq S_A \times S_B$$



outputs:  $a, b$  (pure)



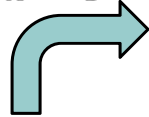
Note that these two states are not reachable.

Synchronous composition

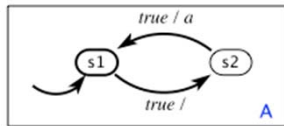
EECS 149/249A, UC Berkeley: 10

## Asynchronous Composition

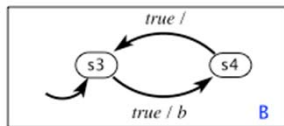
$$S_C \subseteq S_A \times S_B$$



outputs:  $a, b$  (pure)



A



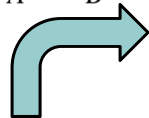
B

Asynchronous composition using interleaving semantics

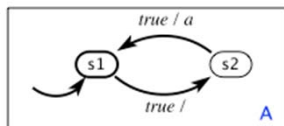
EECS 149/249A, UC Berkeley: 11

## Asynchronous Composition

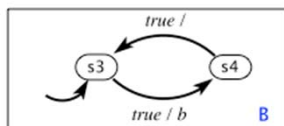
$$S_C \subseteq S_A \times S_B$$



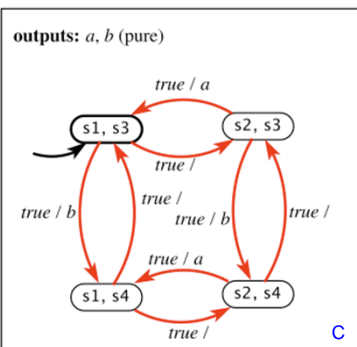
outputs:  $a, b$  (pure)



A



B



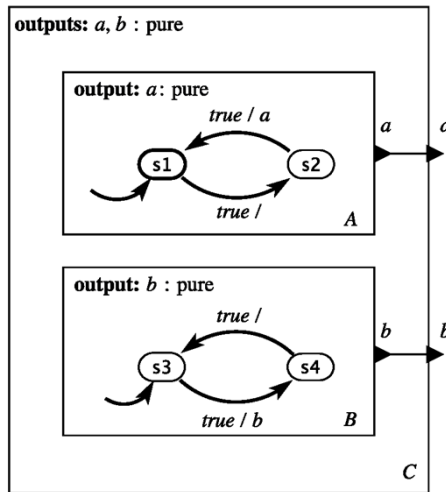
C

Note that now all states are reachable.

Asynchronous composition using interleaving semantics

EECS 149/249A, UC Berkeley: 12

## Syntax vs. Semantics

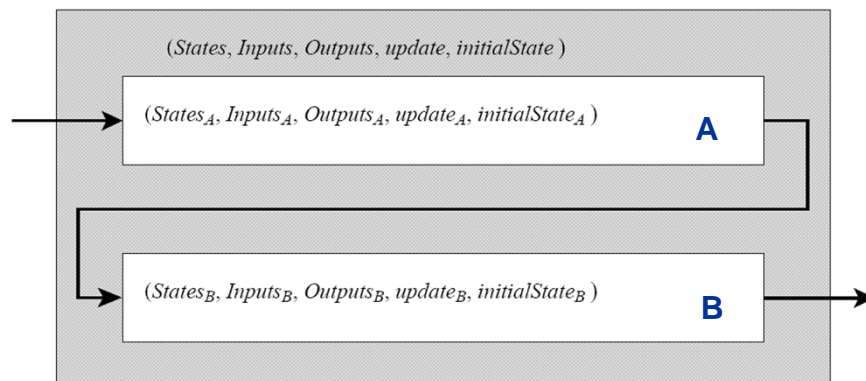


Synchronous  
or  
Asynchronous  
composition?

If asynchronous,  
does it allow  
simultaneous  
transitions in  
A & B?

EECS 149/249A, UC Berkeley: 13

## Cascade Composition



Output port(s) of A connected to input port(s) of B

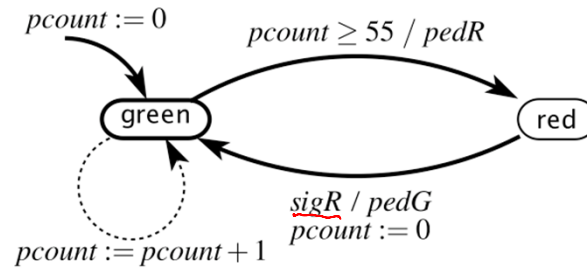
EECS 149/249A, UC Berkeley: 14

## Example: Pedestrian Light

**variable:**  $pcount: \{0, \dots, 55\}$

**input:**  $sigR$ : pure

**outputs:**  $pedG, pedR$ : pure



This light stays green for 55 seconds, then goes red.  
Upon receiving a sigR input, it repeats the cycle.

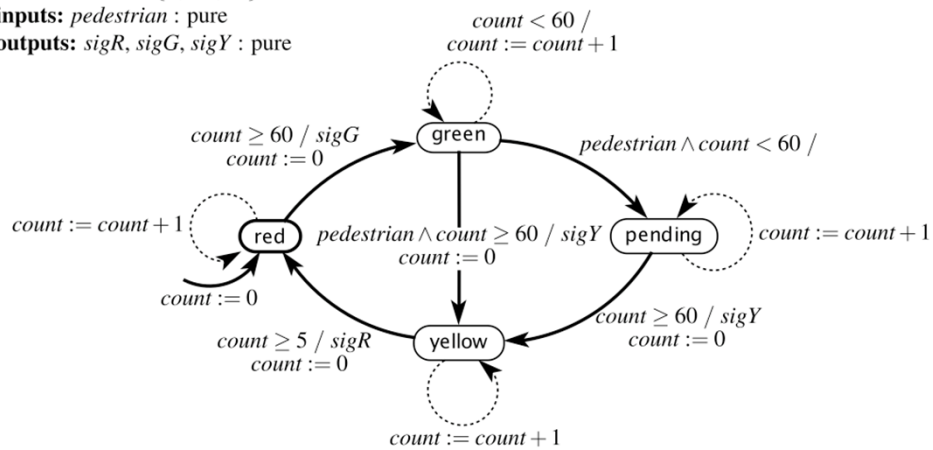
EECS 149/249A, UC Berkeley: 15

## Example: Car Light

**variable:**  $count: \{0, \dots, 60\}$

**inputs:**  $pedestrian$ : pure

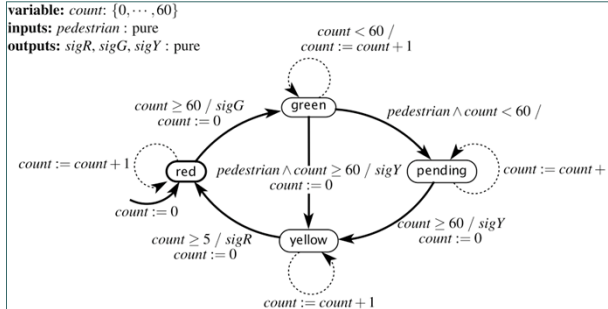
**outputs:**  $sigR, sigG, sigY$ : pure



EECS 149/249A, UC Berkeley: 16



# Pedestrian Light with Car Light

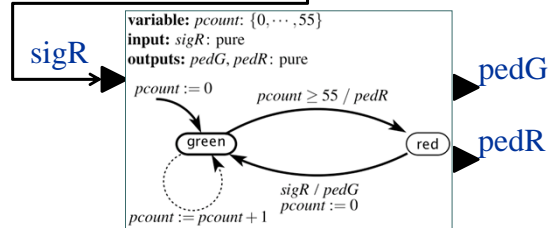


sigY

sigG

sigR

What is the size of the state space of the composite machine?

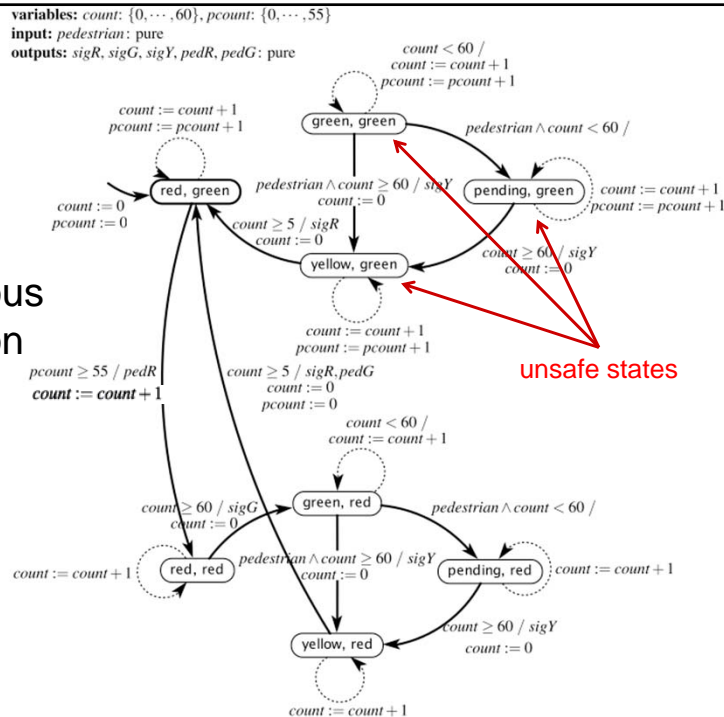


pedG

pedR

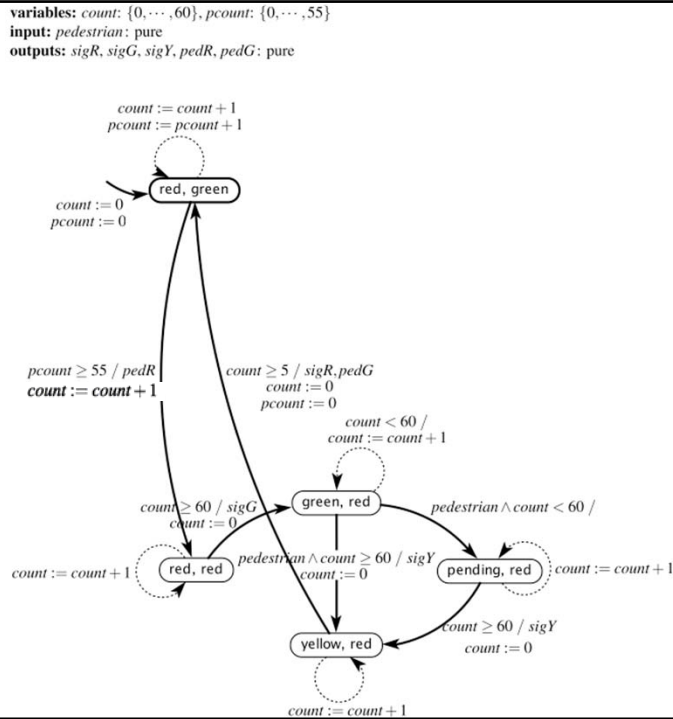
EECS 149/249A, UC Berkeley: 17

# Synchronous composition

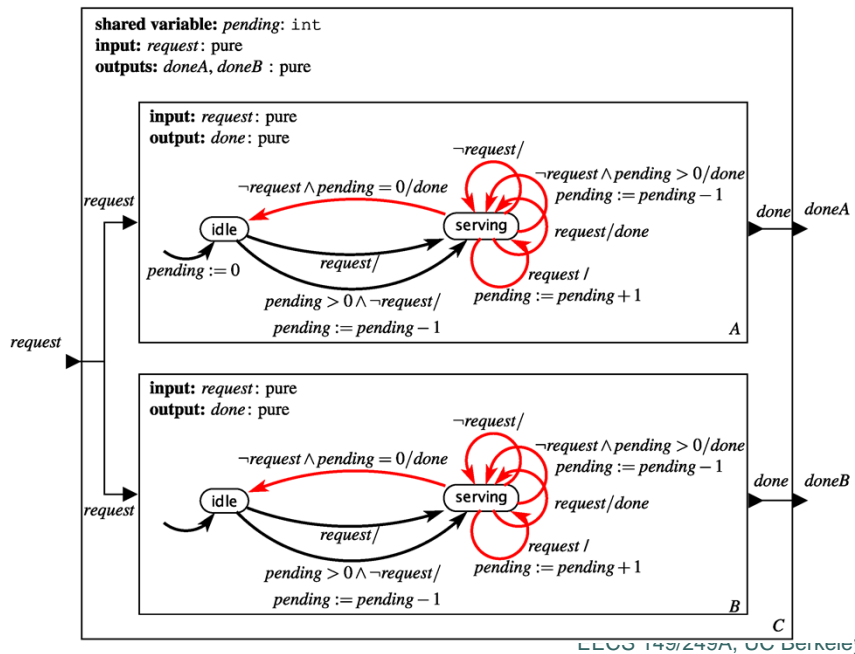


unsafe states

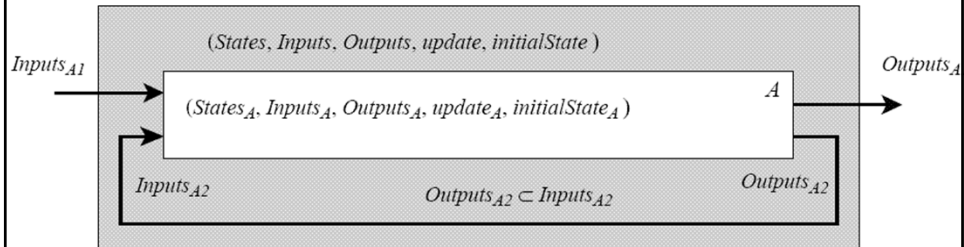
Synchronous composition with unreachable states removed



### Shared Variables: Two Servers



## Feedback Composition



Reasoning about feedback composition can be very subtle.  
(this topic is out of scope for EECS149/249A)