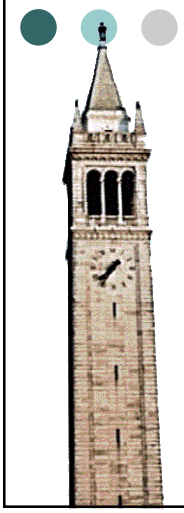


Introduction to Embedded Systems

Edward A. Lee & Sanjit A. Seshia
UC Berkeley
EECS 149
Spring 2009

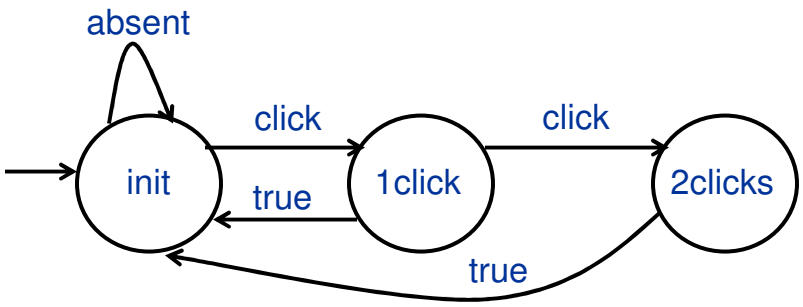
Lecture 6: Hybrid Systems, Part II

Material drawn from notes by R. Alur, P. Bouyer, C. Tomlin

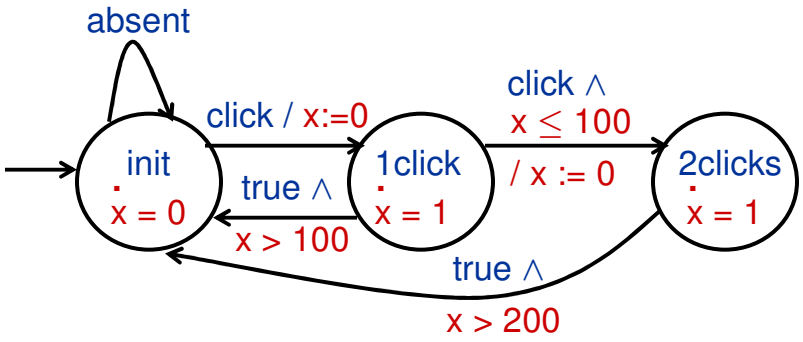
Topic of this Lecture

- Timed Automata
  - sub-class of Hybrid Automata useful for modeling real-time systems

Example: Capturing a “Double-Click” of a Mouse with a FSM



Example: Capturing a “Double-Click” with a Timed Automaton



Recall: Formal Representation of Hybrid Automaton

A hybrid automaton is a tuple: $(Q, X, \Sigma, U, Init, F, J, Inv)$

Q	finite set of modes
X	finite set of continuous state variables $\{x_1, x_2, \ldots, x_n\}$, $x_i \in \mathbb{R}$
Σ	set of discrete input symbols
U	set of continuous input signals, $\{u_1, u_2, \ldots, u_k\}$, $u_i \in \mathbb{R}$
$Init$	initial condition, $Init \subseteq Q \times \mathbb{R}^n$
F	flows, defining differential equations for each variable in each mode
J	jumps, $J : Q \times Guards \rightarrow Q \times Resets$ where an element of $Guards$ is a subset of $\Sigma \times \mathbb{R}^n \times \mathbb{R}^k$, and $Resets$ is a set of assignments of the form $x_i := expr(X, U)$
Inv	mode invariant, mapping a state to the subspace of $\mathbb{R}^n$ in which the X variables can take values

For a timed automaton: x_i's called "clock variables"

All flows are of the form $\dot{x}_i = c$, c a constant

All guards are sets of constraints of the form

$x_i \geq c$ or $x_i \leq c$, $c \in \mathbb{Q}$

All resets are of the form $x_i := 0$

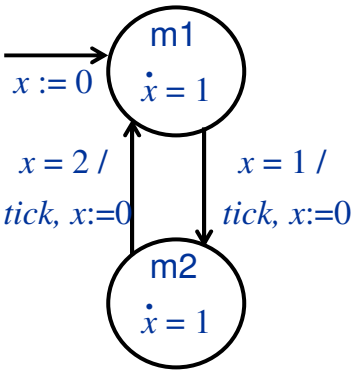
Flavors of Timed Automata

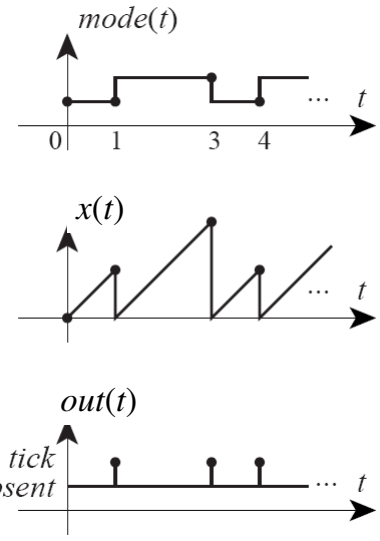
- Classic Timed Automata
  - RHS of all differential equations is 1
  - Single-speed clock that precisely tracks real time
- Multi-rate Automata
  - Can have clocks of different speeds

Applications of Timed Automata

- Real-time controllers
- Self-timed circuits
- Network protocols with timing-dependent behavior
- Scheduling of jobs

Example: A 'Tick' Generator





What does $x(t)$ look like?