Introduction to Embedded Systems

Sanjit A. Seshia
UC Berkeley
EECS 149/249A
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Chapter 15: Reachability Analysis and Model Checking

The Challenge of Dependable Software in Embedded Systems

Today's medical devices run on software… software defects can have life-threatening consequences.

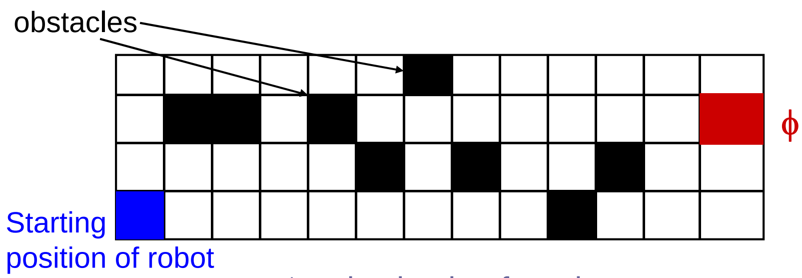
[Journal of Pacing and Clinical Electrophysiology, 2004]

[different device]

"the patient collapsed while walking towards the cashier after refueling his car […] A week later the patient complained to his physician about an increasing feeling of unwell-being since the fall."

"In 1 of every 12,000 settings, the software can cause an error in the programming resulting in the possibility of producing paced rates up to 185 beats/min."

A Robot delivery service, with obstacles



ϕ = destination for robot

Specification:

The robot eventually reaches ϕ

Suppose there are n destinations $\phi_1, \phi_2, \ldots, \phi_n$

The new specification could be that

The robot visits $\phi_1, \phi_2, \ldots, \phi_n$ in that order

Reachability Analysis and Model Checking

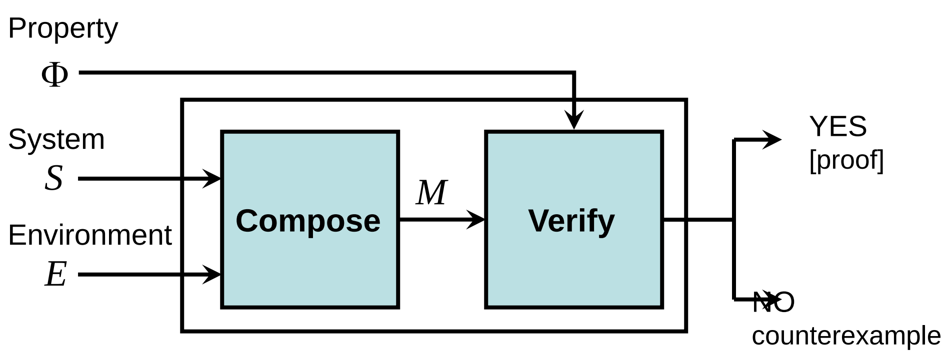
Reachability analysis is the process of computing the set of reachable states for a system.

- preceding problems can be solved using reachability analysis

Model checking is an algorithmic method for determining if a system satisfies a formal specification expressed in temporal logic.

Model checking typically performs reachability analysis.

Formal Verification



Open vs. Closed Systems

A closed system is one with no inputs

(a) Open system (b) Closed system

For verification, we obtain a closed system by composing the system and environment models

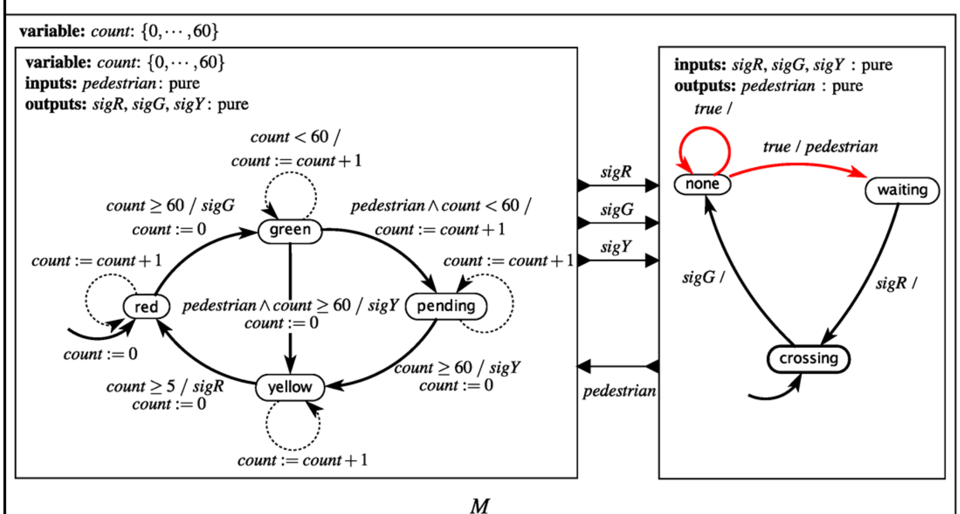
Model Checking G *p*

Consider an LTL formula of the form **G**p where p is a proposition   (p is a property on a single state)

To verify **G**p on a system M, one simply needs to enumerate all the reachable states and check that they all satisfy p.

Traffic Light Controller Example

Property: $G(\neg(green \wedge crossing))$



Model Checking G *p*

Consider an LTL formula of the form **G**p where p is a proposition (p is a property on a single state)

To verify **G**p on a system M, one simply needs to enumerate all the reachable states and check that they all satisfy p.

The state space found is typically represented as a directed graph called a state graph.

When M is a finite-state machine, this reachability analysis will terminate (in theory).

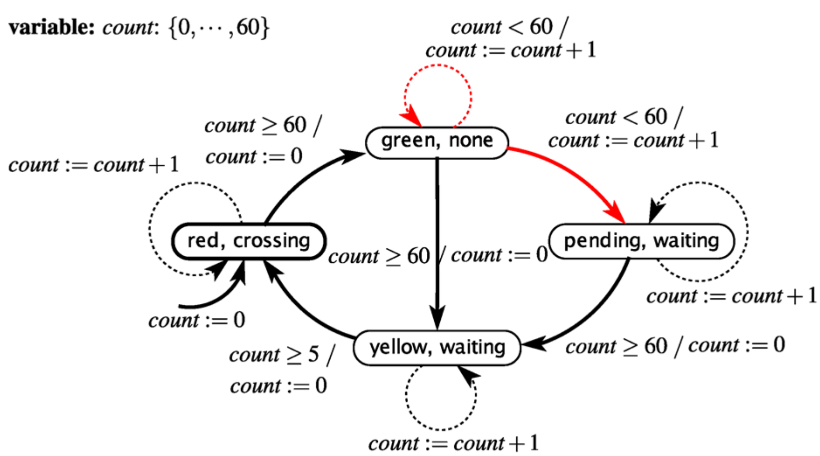
In practice, though, the number of states may be prohibitively large consuming too much run-time or memory (the state explosion problem).

Composed FSM for Traffic Light Controller

Property: $G(\neg(green \wedge crossing))$

This FSM has 189 states

(accounting for different values of count)



Reachability Analysis Through Graph Traversal

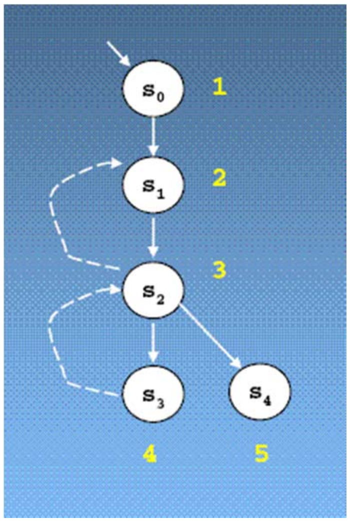
Construct the state graph on the fly.

Start with initial state, and explore next states using a suitable graph-traversal strategy.

Depth-First Search (DFS)

Maintain 2 data structures:

1. Set of visited states R
2. Stack with current path from the initial state



Potential problems for a huge graph?

Generating counterexamples

If the DFS algorithm finds the target (‘error’) state s, how can we generate a trace from the initial state to that state?

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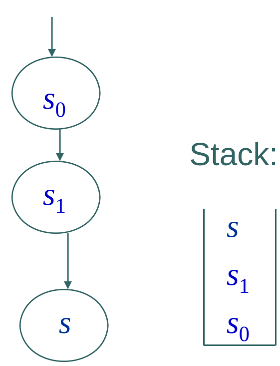
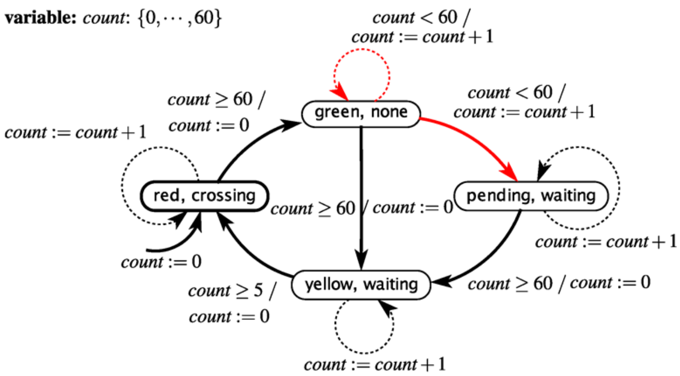


Simply read the trace off the stack

Explicit State Model Checking Example

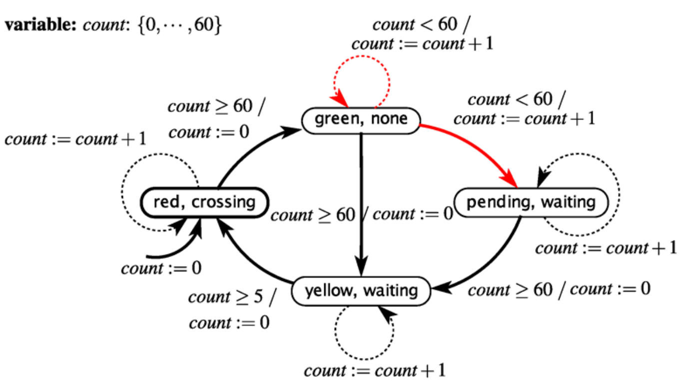
Property: $G(\neg(green \wedge crossing))$



R = { (red, crossing, 0) }

Explicit State Model Checking Example

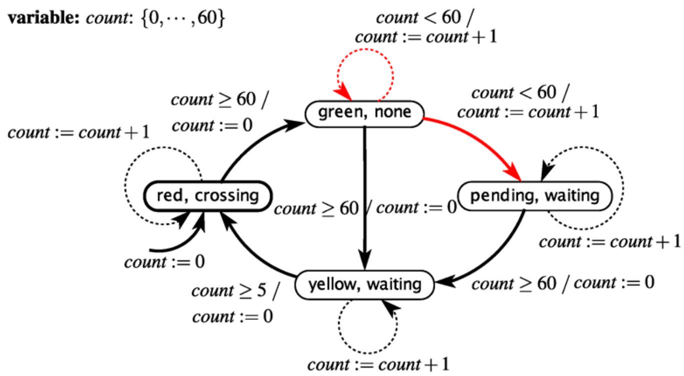
Property: $G(\neg(green \wedge crossing))$



R = { (red, crossing, 0), (red, crossing, 1) }

Explicit State Model Checking Example

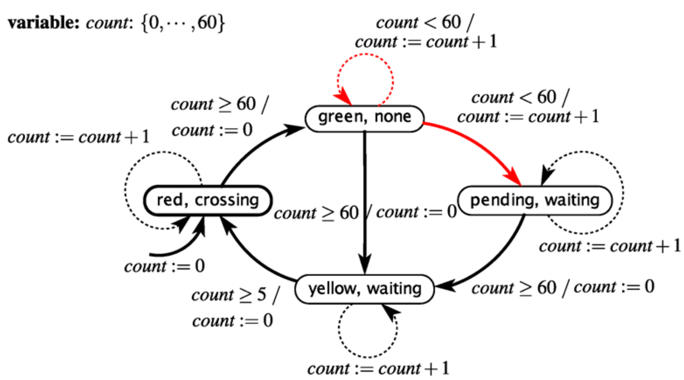
Property: $G(\neg(green \land crossing))$



R = { (red, crossing, 0), (red, crossing, 1), … (red, crossing, 60) }

Explicit State Model Checking Example

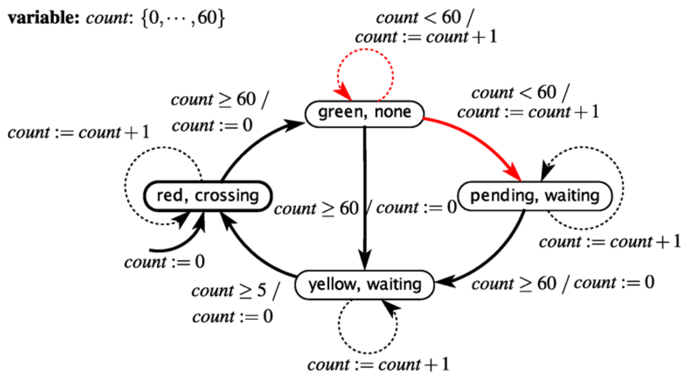
Property: $G(\neg(green \land crossing))$



R = { (red, crossing, 0), (red, crossing, 1), … (red, crossing, 60), (green, none, 0) }

Explicit State Model Checking Example

Property: $G(\neg(green \wedge crossing))$



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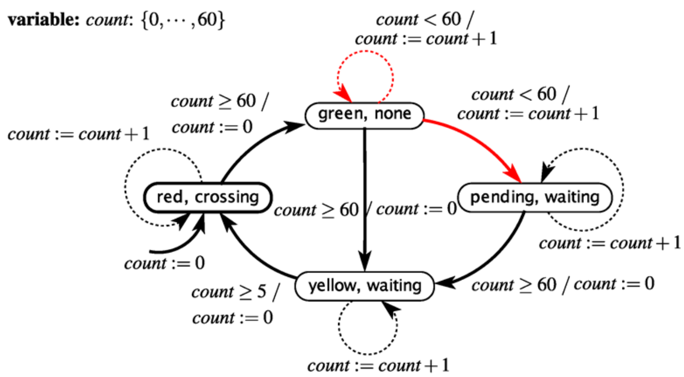

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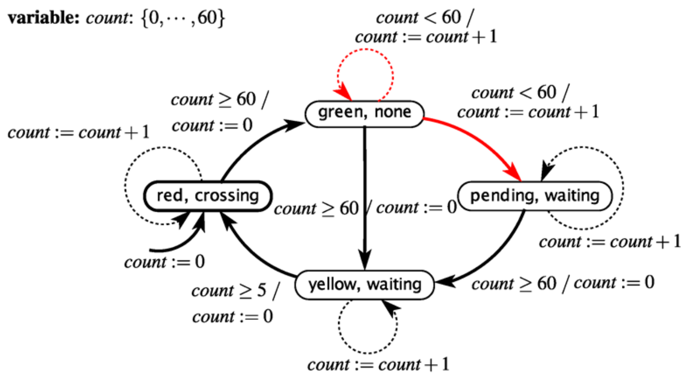

R = { (red, crossing, 0), (red, crossing, 1), … (red, crossing, 60),
(green, none, 0), (green, none, 1), …, (green, none, 60),
(yellow, waiting, 0) }

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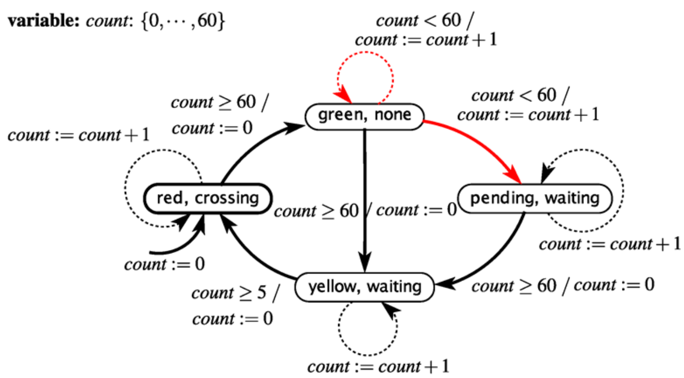
Explicit State Model Checking Example

Property: $G(\neg(green \wedge crossing))$



R = { (red, crossing, 0), (red, crossing, 1), … (red, crossing, 60),
(green, none, 0), (green, none, 1), …, (green, none, 60),
(yellow, waiting, 0), … (yellow, waiting, 5) }

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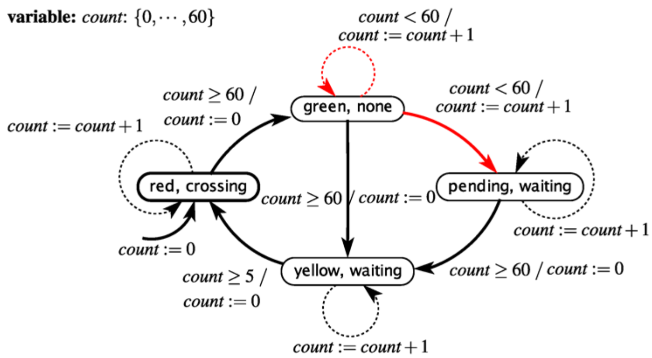
Explicit State Model Checking Example

Property: $G(\neg(green \wedge crossing))$



R = { (red, crossing, 0), (red, crossing, 1), … (red, crossing, 60),
(green, none, 0), (green, none, 1), …, (green, none, 60),
(yellow, waiting, 0), … (yellow, waiting, 5),
(pending, waiting, 1), …, (pending, waiting, 60) }

The *Symbolic* Approach

Rather than exploring new reachable states one at a time, we can explore new sets of reachable states

- However, we only represent sets implicitly, as Boolean functions

Set operations can be performed using Boolean algebra

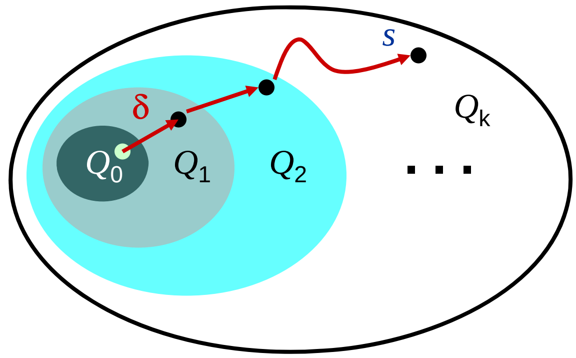
Represent a finite set of states S by its characteristic Boolean function f_S

- $f_S(x) = 1$ iff $x \in S$

Similarly, the state transition function δ yields a set δ(*s*) of next states from current state *s*, and so can also be represented using a characteristic Boolean function for each *s*.

Symbolic Approach (Breadth First Search)

- Generate the state graph by repeated application of transition function (δ)
- If the goal state reached, stop & report success. Else, continue until all states are seen.



The Symbolic Reachability Algorithm

Input : Initial state s_0 and transition relation δ for closed finite-state system M, represented symbolically

Output: Set R of reachable states of M, represented symbolically

Initialize: *Current set of reached states* $R = \{s_0\}$

Symbolic_Search() $\{$

$R_{\text{new}} = R$

while $R_{new} \neq \emptyset$ **do**

  $R_{\text{new}} := \{s' \mid \exists s \in R \text{ s.t. } s' \in \delta(s)\} \setminus R$

  $R := R \cup R_{\text{new}}$

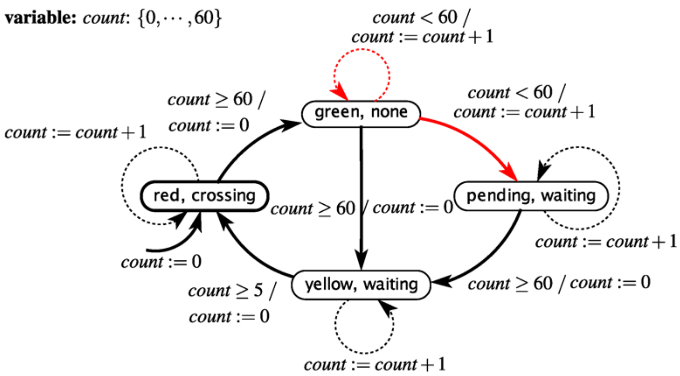
end

$\}$

Two extremely useful techniques:
Binary Decision Diagrams (BDDs)
Boolean Satisfiability (SAT)
These are covered in EECS 144

Symbolic Model Checking Example

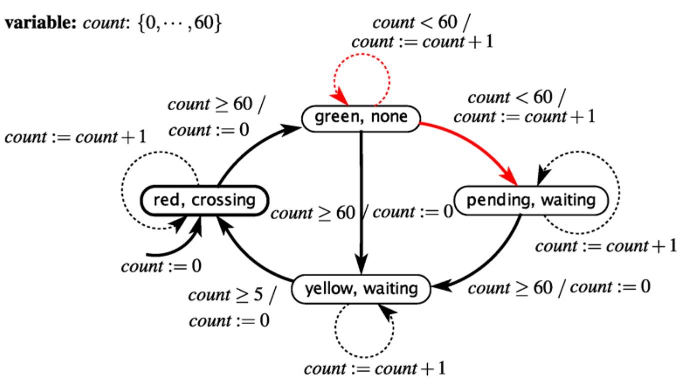
Property: $G(\neg(green \wedge crossing))$



R, set of reachable states, represented by: $(v_1 = \text{red} \wedge v_2 = \text{crossing} \wedge \text{count} = 0)$

Symbolic Model Checking Example

Property: $G(\neg(green \wedge crossing))$



R, set of reachable states, represented by: $(v_1 = \text{red} \wedge v_2 = \text{crossing} \wedge 0 \leq \text{count} \leq 1)$

Symbolic Model Checking Example

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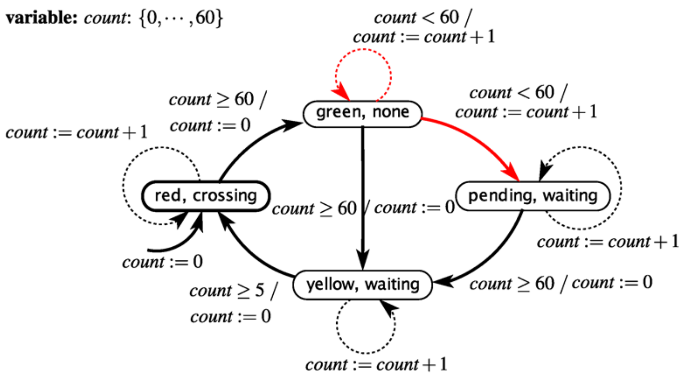

R, set of reachable states, represented by:

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Symbolic Model Checking Example

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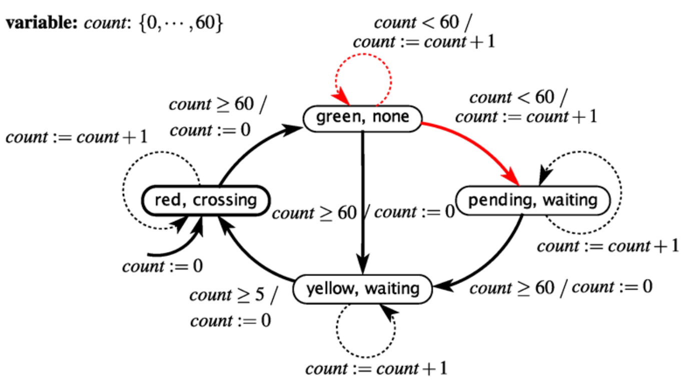

R, set of reachable states, represented by:

$$(v_1 = \text{red} \wedge v_2 = \text{crossing} \wedge 0 \leq \text{count} \leq 60)$$
$$\vee (v_1 = \text{green} \wedge v_2 = \text{none} \wedge \text{count} = 0)$$

Symbolic Model Checking Example

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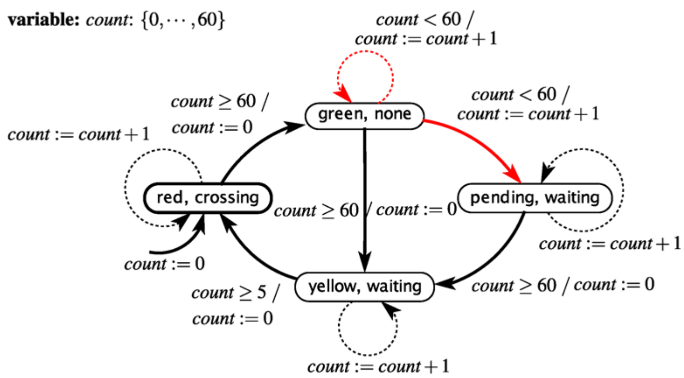
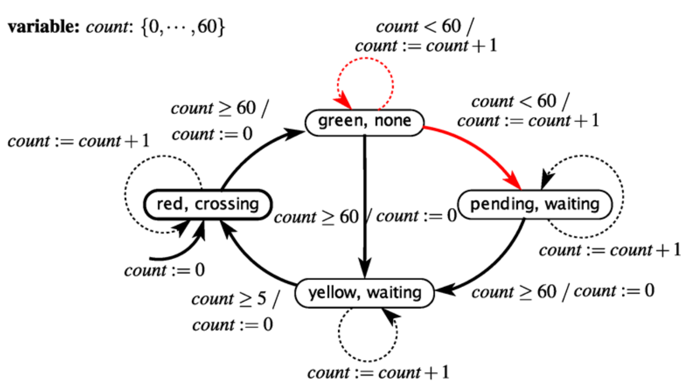


R, set of reachable states, represented by:

$$
\begin{aligned}
&(v_1 = \text{red} \wedge v_2 = \text{crossing} \wedge 0 \leq \text{count} \leq 60) \\
&\vee (v_1 = \text{green} \wedge v_2 = \text{none} \wedge 0 \leq \text{count} \leq 1) \\
&\vee (v_1 = \text{pending} \wedge v_2 = \text{waiting} \wedge \text{count} = 1)
\end{aligned}
$$

Symbolic Model Checking Example

Property: $G(\neg(green \wedge crossing))$



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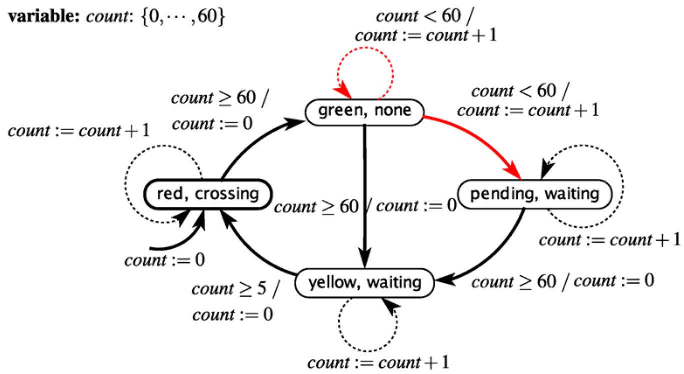

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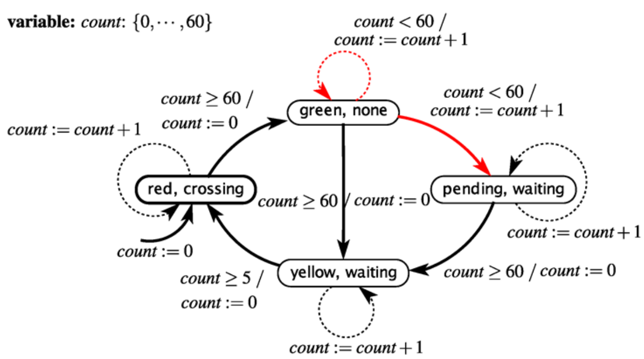
Symbolic Model Checking Example

Property: $G(\neg(green \wedge crossing))$



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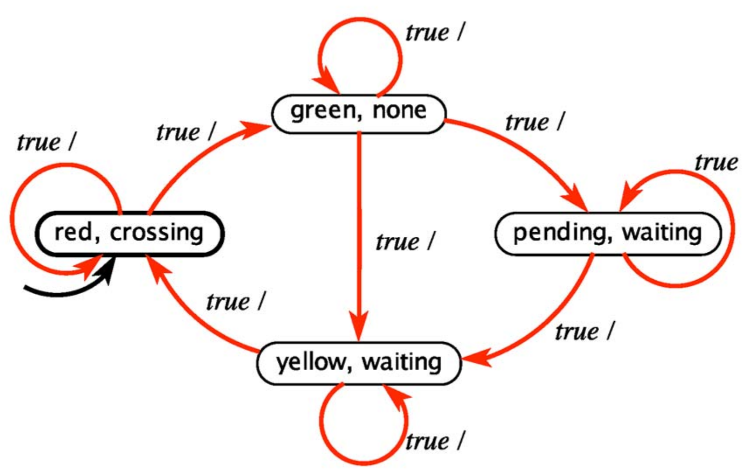
Abstraction in Model Checking

- Should use simplest model of a system that provides proof of safety.
- Simpler models have smaller state spaces and easier to check.
- The challenge is to know what details can be abstracted away.
- A simple and useful approach is called *localization abstraction*.
- A localization abstraction hides state variables that are irrelevant to the property being verified.

Abstract Model for Traffic Light Example

Property: $G(\neg(green \wedge crossing))$

What's the hidden variable?



Model Checking Liveness Properties

- A **safety** property (informally) states that "nothing bad ever happens" and has finite-length counterexamples.
- A **liveness** property, on the other hand, states "something good eventually happens", and only has infinite-length counterexamples.
- Model checking liveness properties is more involved than simply doing a reachability analysis. See Section 15.4 for more information.

Suppose we have a Robot that must pick up multiple things, in any order

$$\phi_i = \text{robot picks up item } i, \text{ where } 1 \le i \le n$$

How would you state this goal in temporal logic?

Suppose we have a Robot that must pick up multiple things, in any order

ϕ_i = robot picks up item i, where $1 \leq i \leq n$

Goal to be achieved is:

$$\mathbf{F}\phi_1 \wedge \mathbf{F}\phi_2 \wedge \cdots \wedge \mathbf{F}\phi_n$$

Variant: Suppose we have a Robot that must pick up multiple things, ***in a specified order***

ϕ_i = robot picks up item i, where $1 \leq i \leq n$

How would you state this goal in temporal logic?

Controller Synthesis

$$\phi_i = \text{robot picks up item } i \text{, where } 1 \leq i \leq n$$

Goal to be achieved is:

$$\mathbf{F}(\phi_1 \wedge \mathbf{F}(\phi_2 \wedge \cdots \wedge \mathbf{F}\phi_n))$$

Consider the first part alone:

$$\mathbf{F}(\phi_1)$$

How can we use model checking to synthesize a control strategy?

Controller Synthesis

Recall that:

$$\mathbf{F}(\phi_1) = \neg\mathbf{G}(\neg\phi_1)$$

Therefore, we can construct a counterexample to:

$$\mathbf{G}(\neg\phi_1)$$

The counterexample is a trace that gets the robot to the desired point.