Societal need for [traffic] information systems

Rough estimates of congestion impacts
- 4.2 billion hours extra travel in the US
- Accounts for 2.9 billion gallons of fuel
- Congestion cost of 78 billion dollars

[2007 Urban Mobility Report, September 2007, Texas Transportation Institute, David Schrank & Tim Lomax]

Traffic information systems
- Call in numbers (511)
- Changeable message signs (CMS)
- Online navigation devices
- Web-based commuter services:
  - www.511.org
  - www.traffic.com
  - Google Traffic
- Cellular phone web browsing
- Connected aftermarket devices
Source of today’s traffic information

**Dedicated traffic monitoring infrastructure:**
- Self inductive loops
- Wireless pavement sensors
- FasTrak, EZ-pass transponders
- Cameras
- Radars
- License plate readers

**Issues of today’s dedicated infrastructure**
- Installation costs
- Maintenance costs
- Reliability
- Coverage
- Privacy intrusion

The next battle of traffic information systems

**Secondary network coverage**
- Expressways
- Side roads
- Arterials
- Rural roads

**Scientific challenges**
- Varying penetration rate
- Proper traffic models based on accurate mapping information
- Reliability of the traffic estimates
Web 2.0 on wheels

Emergence of the mobile internet
- Internet accesses from mobile devices skyrocketing
- Mobile devices outnumber PCs by 5:1
- 1.5 million devices/day (Nokia)
- Redefining the mobile market: Google, Apple, Nokia, Microsoft, Intel, IBM, etc.
- Open source computing: Symbian Foundation, Android, Linux

Sensing and communication suite
- GSM, GPRS, WiFi, bluetooth, infrared
- GPS, accelerometer, light sensor, camera, microphone

Smartphones and Web 2.0
- Context awareness
- Sensing based user generated content

Outline

1. Traffic information systems
   1. Existing dedicated traffic monitoring infrastructure
   2. Web 2.0 on wheels

2. Mobile Millennium
   1. System
   2. Privacy aware sampling

3. Inverse modeling and data assimilation
   1. A short introduction to traffic modeling
   2. The Moskowitz Hamilton-Jacobi equation
   3. Internal boundary conditions using the inf-morphism property
   4. Data assimilation in a privacy aware environment
   5. Mobile Century (February 8th, 2008)

4. The launch of Mobile Millennium
   1. The Bay Area
   2. New York
Ubiquitous traffic monitoring cyberinfrastructure

Sensing
- Millions of mobile devices as new sources for data

Communication
- Existing cell phone infrastructure to collect raw data and receive traffic information

Data assimilation
- Real-time, online traffic estimation

Privacy Management
- Encrypted transactions
- Client authentication
- Data anonymization

Privacy issues for location based services
Spatially aware traffic monitoring

**Virtual Trip Lines (VTLs)**
- GPS coordinates defined geographic markers (virtual loop detectors)
- Deployed at privacy aware locations
- Trigger GPS updates for a proportion of the phones crossing them.
- Phone anonymizes data
- GPS update is encrypted and sent

[Hoh et al. Mobisys, 2008]

Nationwide deployment of a virtual infrastructure

Spatially aware sampling
Sampling of the arterial (and the highway) traffic done using Virtual Trip Lines (VTLs)
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A very brief introduction to traffic flow modelling

Seminal hydrodynamic model: the Lighthill-Whitham-Richards partial differential equation
- Nonlinear first order hyperbolic scalar conservation law
- Concave flux function (empirical fundamental diagram)
- Weak boundary conditions

\[ \frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0 \]

\( \rho(x, t) \) is the vehicle density.
A very brief introduction to traffic flow modelling

**Most basic fundamental diagram – Greenshields flux function**
- Velocity modelled as a linear function of density:
  \[
  v(\rho) = v_{\text{max}} \left( 1 - \frac{\rho}{\rho_{\text{max}}} \right)
  \]
- Corresponding flux is parabolic:
  \[
  q(\rho) = \rho v_{\text{max}} \left( 1 - \frac{\rho}{\rho_{\text{max}}} \right)
  \]
  \[
  \frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0
  \]

Numerous other diagrams used in this work:
- Parabolic
- Triangular
- Trapezoidal
- Hybrid
- Tong
- ...

Challenges of nonlinearity

**Well posed Cauchy problem**

First order scalar nonlinear hyperbolic PDE (LWR PDE)

\[
\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0
\]

Initial conditions

\[
\rho(x, 0) = \rho_0(x)
\]

**Strong boundary conditions**: not well posed

\[
\rho(a, t) = \rho_a(t) \quad \rho(b, t) = \rho_b(t)
\]
 Challenges of nonlinearity

**Well posed Cauchy problem**

First order hyperbolic PDE (LWR PDE)

\[
\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0
\]

Initial conditions

\[\rho(x, 0) = \rho_0(x)\]

**Weak boundary conditions**: necessary for well-posedness

\[
\begin{cases}
\rho(a, t) = \rho_a(t) \text{ or } \\
q'(\rho(a, t)) \leq 0 \text{ and } q'(\rho_a(t)) \leq 0 \text{ or } \\
q''(\rho(a, t)) \leq 0 \text{ and } q''(\rho_a(t)) \geq 0 \text{ and } q(\rho(a, t)) \leq q(\rho_a(t)) \\
\rho(b, t) = \rho_b(t) \text{ or } \\
q'(\rho(b, t)) \geq 0 \text{ and } q'(\rho_b(t)) \geq 0 \text{ or } \\
q''(\rho(b, t)) \geq 0 \text{ and } q''(\rho_b(t)) \leq 0 \text{ and } q(\rho(b, t)) \leq q(\rho_b(t))
\end{cases}
\]

[Lighthill-Whitham, 1955; Richards, 1956; Bardos Leroux Nedelec, 1979; Strub, Bayen 2006]

---

**Nonlinear features to capture for acceptable modeling accuracy**

- Spillovers, moving bottlenecks
- Shockwaves, expansion waves
Nonlinear features to capture for acceptable modeling accuracy

- Spillovers, moving bottlenecks
- Shockwaves, expansion waves

Challenges of nonlinearity

Nonlinear and nonsmooth features are necessary to capture the physics with acceptable modeling accuracy

- Spillovers, moving bottlenecks
- Shockwaves, expansion waves

Hydrodynamic models (PDEs)

- Algebraically compact
- Numerically tractable
- Accurate abstraction of traffic
- Can be used for control and estimation purposes
- Pose challenging theoretical problems
Challenges of nondifferentiability

Nonsmooth features of traffic are captured by nonsmooth discretization

- Godunov numerical schemes (max/min operators)
- Boolean logic in the implementation of boundary conditions (in the form of ghost cells)

\[ \rho_i^{n+1} = \rho_i^n - r(q_G(\rho_i^n, \rho_{i+1}^n) - q_G(\rho_{i-1}^n, \rho_i^n)) \]

\[ q_G(\rho_1, \rho_2) = \begin{cases} 
q(\rho_2) & \text{if } \rho_c < \rho_2 < \rho_1 \\
q(\rho_c) & \text{if } \rho_2 < \rho_c < \rho_1 \\
q(\rho_1) & \text{if } \rho_2 < \rho_1 < \rho_c \\
\min(q(\rho_1), q(\rho_2)) & \text{if } \rho_1 \leq \rho_2 
\end{cases} \]
Challenges of nondifferentiability

Nonsmooth features of traffic are captured by nonsmooth discretization

- Godunov numerical schemes (max/min operators)
- Boolean logic in the implementation of boundary conditions (in the form of ghost cells)

\[ \rho^{n+1} = \mathcal{F}(\rho^n) + \mathcal{G}(u^n) \]

- Dynamics is nonlinear and nonsmooth
- It is required to capture the proper entropy solution of the PDE

Proper treatment of network junctions introduces additional nonlinearity

- Network junctions are nonsmooth (weak boundary conditions)
- The solution needs to be computed iteratively by solving a linear program at each time step

\[ (\rho_1^{n+1}, \rho_2^{n+1}, \rho_3^{n+1}) = \mathcal{L} \mathcal{P}(\rho_1^n, \rho_2^n, \rho_3^n) \]

- Solution of a LP is not continuous or smooth in the coefficients of the LP

Data assimilation / inverse modeling

How to incorporate Lagrangian (trajectory based) and Eulerian (control volume based) measurements in a flow model.

Contributions

Modeling

- Weak BC for the LWR- \( p \) model
  [Strub, Bayen, IJRNC, 2006]
- Hamilton-Jacobi equation for Cumulated vehicle number
  [Aubin, Bayen, Saint-Pierre, SIAM SICON, 2008]
- Second order models (PDE systems) for highway traffic
  [Blandin, Work, Gaotin, Piccoli, Bayen, subm. SIAM SIAP, 2008]
- First order velocity model (PDE systems) for highway traffic
  [Work, Tossavainen, Tracton, Bayen, CDC 2008]

Data assimilation

- Newtonian relaxation
  [Herrera, Bayen, subm. Transportation Research B, 2008]
- Ensemble Kalman filtering
  [Work, Tossavainen, Jacobson, Bayen, ACC 2009]
Contributions

Hamilton-Jacobi framework
- Integration of internal boundary conditions
- Explicit solutions for piecewise affine conditions
- Linear programming solutions of robust estimation problems
  [Claudel, Bayen, Allerton CCC, 2009]

Systems and data work
- Mobile Century data assimilation and analysis
  [Herrera, Work, Bayen, subm. Transportation Research C, 2009]
- Systems architecture
  [Hoh, et al., Mobisys 2008]

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The Moskowitz function

Main problem: link an Eulerian representation of traffic (conservation of vehicle based) with Lagrangian (trajectory based) sensing

- Cumulative vehicle number (label)
  \[ M(t, x) \]

- Satisfies a Hamilton-Jacobi equation
  \[
  \frac{\partial M}{\partial t} - q \left( - \frac{\partial M}{\partial x} \right) = 0
  \]

- With boundary condition(s)
  \[ M(t, \chi) \]
  \[ M(t, \xi) \]

- And initial conditions
  \[ M(0, x) \]
Physical interpretation of the Moskowitz function

17th vehicle which entered the link of highway has driven exactly 2.5 miles in 1 minute

\[ M(t,x) = 17 \quad t = 1 \text{ min} \quad x = 2.5 \text{ miles} \]

Physical interpretation of the initial condition

Vehicle entering at t=0 is labeled zero (arbitrary). If value at 3.2 miles at t=0 is -25, there are 25 cars between x=0 and x=3.2

\[ M(0,x) = -25 \quad x = 3.2 \text{ miles} \]

Labels of the vehicles at initial time

\[ M(0,x) = M_0(0,x) \]
Physical interpretation of the boundary condition

47 vehicles entered between \( t=0 \) and \( t=2.5 \)

\[ M(t, \xi) = 47 \]

Boundary condition:

\[ M(t, \xi) = \gamma(t, \xi) \]

Physical interpretation of the level sets

Level sets of the Moskowitz function correspond to trajectories

\[ M(t, x) \]

\((t,x)\) belongs to the trajectory of vehicle #3 \( \Leftrightarrow \) \( M(t,x)=3 \)
Experimentally measured Moskowitz surface

Level sets of the Moskowitz function correspond to trajectories

Full Cauchy problem

\[ \frac{\partial M}{\partial t} - q \left( \frac{\partial M}{\partial x} \right) = 0 \]

\[ M(t, x) \mathbf{M}(t, \chi) = \beta(t, \chi) \]

\[ M(0, x) = M_0(0, x) \]

\[ M(t, \xi) = \gamma(t, \xi) \]
Full Cauchy problem

PDE, left boundary condition, right boundary condition, initial condition

\[ \frac{\partial M}{\partial t} - q \left( - \frac{\partial M}{\partial x} \right) = 0 \]

\[
\begin{align*}
M(0, x) &= M_0(0, x) \quad \forall x \in X \\
M(t, \xi) &= \gamma(t, \xi) \quad \forall t \in \mathbb{R}_+ \\
M(t, \chi) &= \beta(t, \chi) \quad \forall t \in \mathbb{R}_+
\end{align*}
\]

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Epigraphical characterization of the solution

Idea: characterize the Moskowitz surface as the lower envelope of a capture basin

Let us consider the epigraphs of the boundary of the domain

Reminder: definition of the capture basin

The capture basin of a target with a constraint set is the set of points which can reach the target in finite time while staying in the constraint set

\[ \dot{x}(t) \in F(x(t)) \]

Capt\(_F\)(\(\mathcal{K}, \mathcal{C}\)) := \{ X_0 \in \mathcal{K} \mid \exists X(\cdot) \in S_F(X_0) \text{ and } \exists T \geq 0 \text{ such that } X(T) \in \mathcal{C} \text{ and } \forall t \in [0, T], X(t) \in \mathcal{K} \} \]
Reminder: definition of the capture basin

The capture basin of a target with a constraint set is the set of points which can reach the target in finite time while staying in the constraint set.

\[
\dot{x}(t) \in F(x(t)) \\
K \quad X_0 \quad C
\]

\[\text{Capt}_F(K, C) := \{X_0 \in K \mid \exists X(\cdot) \in S_F(X_0) \text{ and } \exists T \geq 0 \\
\text{such that } X(T) \in C \text{ and } \forall t \in [0, T], \ X(t) \in K\}\]
Fundamental union property of capture basins

The capture basin of a finite union of targets is the union of the capture basins of the targets

$$\text{Capt}_F (\mathcal{K}, \bigcup_{i \in I} \mathcal{C}_i) = \bigcup_{i \in I} \text{Capt}_F (\mathcal{K}, \mathcal{C}_i)$$
Fundamental union property of capture basins

The capture basin of a finite union of targets is the union of the capture basins of the targets

\[ \text{Capt}_F(\kappa, C_1 \cup C_2) = \bigcup_{i \in I} \text{Capt}_F(\kappa, C_i) \]

Epigraphical characterization of the solution

Let us consider the epigraphs of the boundary of the domain
Let us consider the epigraphs of the boundary of the domain.
Epigraphical characterization of the solution

Let us consider the epigraphs of the boundary of the domain

Construct an auxiliary dynamics

Consider the following set valued dynamics

\[
F := \begin{cases}
    \tau'(t) = -1 \\
    x'(t) = u(t) \\
    y'(t) = -\varphi^*(u(t))
\end{cases}
\]

where \( u(t) \in \text{Dom}(\varphi^*) \)

Where the Fenchel transform of the Hamiltonian is given by:

\[
\varphi^*(u) = \sup_{p \in \text{Dom}(\psi)} [p \cdot u + q(p)]
\]
In the capture basin? NO

Is it possible to capture the target from a given point?

In the capture basin? YES

Is it possible to capture the target from a given point?
Capture basin has a lower envelope

Viability solution (definition using capture basin)

\[ M(t,x) := \inf_{(t,x,y) \in \text{Capt}_F(K,C)} y \]
The inf-morphism property

The union property for capture basins

\[ \text{Capt}_F \left( \mathcal{K}, \bigcup_{i \in I} \mathcal{C}_i \right) = \bigcup_{i \in I} \text{Capt}_F (\mathcal{K}, \mathcal{C}_i) \]

translates into an inf-morphism property

\[ \forall \ t \geq 0, \ x \in X, \ M_c(t, x) = \inf_{i \in I} M_{c_i}(t, x) \]

Tangential property of the capture basin

This defines a new class of solutions to the HJ PDE:

\[ M(t, x) := \inf_{(t, x, y) \in \text{Capt}_F (\mathcal{K}, \mathcal{C})} y \]

The solution provided by this formula is a lower semicontinuous function. It is the solution to the HJ PDE considered before, in a weaker sense than the viscosity solution. This solution is called the Barron/Jensen-Frankowska (B/J-F) solution.

B/J-F solutions require only the lower semicontinuity of the solution.

In particular: whenever \( M \) is differentiable the tangential properties of the capture basin imply:

\[ \forall (t, x) \in \text{Dom}(M_c) \setminus \text{Dom}(c) \quad \frac{\partial M_c(t, x)}{\partial t} - \psi \left( - \frac{\partial M_c(t, x)}{\partial x} \right) = 0 \]
Adding trajectories is equivalent to adding epigraphs. Very often, drivers drive in violation of the LWR HJ PDE model (because of external disturbances not included in the model: accidents, distraction, excessive speed, etc.). This can be measured:

Now one can continue adding targets in the Form of epigraphs:
Adding trajectories is equivalent to adding epigraphs:

Now one can continue adding targets in the Form of epigraphs:

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Comparing information

What is measured by the outflow detector. Influence of the reading of the inflow detector on what the outflow detector should say.

Data assimilation using linear programming

Initial condition unknown - Δ
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

\[
(i) \quad \inf_{t \in \mathbb{R}_+} (g_\gamma(t,x) - f_\beta(t,x)) \geq \Delta
\]
Data assimilation using linear programming

Initial condition unknown \(-\Delta\)
Left and right boundary conditions known
Internal conditions known, but labels \(M_i\) unknown

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} \left( g_{\gamma}(t, \chi) - f_{\beta}(t, \chi) \right) \geq \Delta
\end{align*}
\]

Condition on the outflow due to the inflow measurement

Data assimilation using linear programming

Initial condition unknown \(-\Delta\)
Left and right boundary conditions known
Internal conditions known, but labels \(M_i\) unknown

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} \left( g_{\gamma}(t, \chi) - f_{\beta}(t, \chi) \right) \geq \Delta
\end{align*}
\]

Reading of our outflow sensor
Data assimilation using linear programming

Initial condition unknown \(-\Delta\)
Left and right boundary conditions known
Internal conditions known, but labels \(M_i\) unknown

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}^+} (g_\gamma(t, x) - f_\beta(t, x)) \geq \Delta
\end{align*}
\]

Comparing information

What is measured by the outflow detector.
Influence of the reading of the inflow detector on what the outflow detector should say

\[M(t, x)\]
Comparing information

Influence of the reading of the outflow detector on what the inflow detector should say

What is measured by the inflow detector.

Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

\[
\begin{align*}
(i) \quad & \inf_{t \in \mathbb{R}_+} (g_\gamma(t,x) - f_\beta(t,x)) \geq \Delta \\
(ii) \quad & \Delta \geq \sup_{t \in \mathbb{R}_+} (-g_\beta(t,\xi) + f_\gamma(t,\xi))
\end{align*}
\]
Adding trajectories is equivalent to adding epigraphs.

Influence of the inflow measurement on the label of the trajectory

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

Data assimilation using linear programming

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} (g_\gamma(t,x) - f_\beta(t,x)) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} (-g_\beta(t,\xi) + f_\gamma(t,\xi)) \\
(iii) & \quad \inf_{t \in [\xi_{min}, \xi_{max}]} (g_\gamma(t,\bar{x}_i(t))) \geq M_i & \forall i \in I
\end{align*}
\]
Data assimilation using linear programming

Initial condition unknown - ∆
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

Label of vehicle $i$ (to be estimated)

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} (g_\gamma(t, x) - f_\beta(t, x)) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} (-g_\beta(t, \xi) + f_\gamma(t, \xi)) \\
(iii) & \quad \inf_{t \in [t_{\min}, t_{\max}]} (g_\gamma(t, \bar{x}_i(t))) \geq M_i \\
\quad \forall i \in I
\end{align*}
\]

Data assimilation using linear programming

Initial condition unknown - ∆
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

Similar condition between outflow and label estimated by the trajectory measurement

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} (g_\gamma(t, x) - f_\beta(t, x)) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} (-g_\beta(t, \xi) + f_\gamma(t, \xi)) \\
(iii) & \quad \inf_{t \in [t_{\min}, t_{\max}]} (g_\gamma(t, \bar{x}_i(t))) \geq M_i \\
(iv) & \quad M_i \geq \sup_{t \in \mathbb{R}_+} (f_\gamma(t, \xi) - g_\alpha(t, \xi)) \\
\quad \forall i \in I
\end{align*}
\]
Data assimilation using linear programming

Initial condition unknown \(-\Delta\)
Left and right boundary conditions known
Internal conditions known, but labels \(M_i\) unknown

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} \left( g_{\gamma}(t, \chi) - f_{\beta}(t, \chi) \right) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} \left( -g_{\beta}(t, \xi) + f_{\gamma}(t, \xi) \right) \\
(iii) & \quad \inf_{t \in [t_{\min}, t_{\max}]} (g_{\gamma}(t, \bar{x}_i(t))) \geq M_i \quad \forall i \in I \\
(iv) & \quad M_i \geq \sup_{t \in \mathbb{R}_+} (f_{\gamma}(t, \xi) - g_{\mu}(t, \xi)) \quad \forall i \in I \\
(v) & \quad \inf_{t \in [t_{\min}, t_{\max}]} (g_{\beta}(t, \bar{x}_i(t))) \geq -\Delta + M_i \quad \forall i \in I \\
(vi) & \quad M_i - \Delta \geq \sup_{t \in \mathbb{R}_+} (f_{\beta}(t, \chi) - g_{\mu_i}(t, \chi)) \quad \forall i \in I
\end{align*}
\]

Adding trajectories is equivalent to adding epigraphs.
Data assimilation using linear programming

Initial condition unknown - $\Delta$
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

$$\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} (g_\gamma(t, x) - f_\beta(t, x)) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} (-g_\beta(t, \xi) + f_\gamma(t, \xi)) \\
(iii) & \quad \inf_{t \in [\tau_{\min}, \tau_{\max}]} (g_\gamma(t, \bar{x}_I(t))) \geq M_i \quad \forall i \in I \\
(iv) & \quad M_i \geq \sup_{t \in \mathbb{R}_+} (f_\gamma(t, \xi) - g_\gamma(t, \xi)) \quad \forall i \in I \\
(v) & \quad \inf_{t \in [\tau_{\min}, \tau_{\max}]} (g_\beta(t, \bar{x}_I(t))) \geq -\Delta + M_i \quad \forall i \in I \\
(vi) & \quad M_i - \Delta \geq \sup_{t \in \mathbb{R}_+} (f_\beta(t, \xi) - g_\beta(t, \xi)) \quad \forall i \in I \\
(vii) & \quad M_j - M_i \geq \sup_{t \in [\tau_{\min}, \tau_{\max}]} (-g_{\mu_j}(t, \bar{x}_I(t))) \quad \forall i \in I, \forall j \in I \setminus \{i\}
\end{align*}$$

Constraint on the label of vehicle $i$ based on the fact that vehicle $j$ has a measured trajectory.

Data assimilation using linear programming

Initial condition unknown - $\Delta$
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

$$\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} (g_\gamma(t, x) - f_\beta(t, x)) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} (-g_\beta(t, \xi) + f_\gamma(t, \xi)) \\
(iii) & \quad \inf_{t \in [\tau_{\min}, \tau_{\max}]} (g_\gamma(t, \bar{x}_I(t))) \geq M_i \quad \forall i \in I \\
(iv) & \quad M_i \geq \sup_{t \in \mathbb{R}_+} (f_\gamma(t, \xi) - g_\gamma(t, \xi)) \quad \forall i \in I \\
(e) & \quad \inf_{t \in [\tau_{\min}, \tau_{\max}]} (g_\beta(t, \bar{x}_I(t))) \geq -\Delta + M_i \quad \forall i \in I \\
(vi) & \quad M_i - \Delta \geq \sup_{t \in \mathbb{R}_+} (f_\beta(t, \xi) - g_\beta(t, \xi)) \quad \forall i \in I \\
(vii) & \quad M_j - M_i \geq \sup_{t \in [\tau_{\min}, \tau_{\max}]} (-g_{\mu_j}(t, \bar{x}_I(t))) \quad \forall i \in I, \forall j \in I \setminus \{i\}
\end{align*}$$
Data assimilation using linear programming

Initial condition unknown - $\Delta$
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

Grey: non linear analytical solution of the Hamilton Jacobi equation. Can be computed explicitly for piecewise affine functions, and semi-explicitly for general nonlinear functions

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+^+} (g_{\gamma}(t, \chi) - f_{\beta}(t, \chi)) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+^+} (-g_{\beta}(t, \xi) + f_{\gamma}(t, \xi)) \\
(iii) & \quad \inf_{t \in [t_{\min}, t_{\max}]} (g_{\gamma}(t, \overline{x}_i(t))) \geq M_i \quad \forall i \in I \\
(iv) & \quad M_i \geq \sup_{t \in \mathbb{R}_+^+} (f_{\gamma}(t, \xi) - g_{\mu_i}(t, \xi)) \quad \forall i \in I \\
(v) & \quad \inf_{t \in [t_{\min}, t_{\max}]} (g_{\beta}(t, \overline{x}_i(t))) \geq -\Delta + M_i \quad \forall i \in I \\
(vi) & \quad M_i - \Delta \geq \sup_{t \in \mathbb{R}_+^+} (f_{\beta}(t, \xi) - g_{\mu_i}(t, \chi)) \quad \forall i \in I \\
(vii) & \quad M_j - M_i \geq \sup_{t \in [t_{\min}, t_{\max}]} (-g_{\mu_j}(t, \overline{x}_i(t))) \\
& \quad \forall i, j \in I \setminus \{i\}
\end{align*}
\]
Data assimilation using linear programming

Initial condition unknown -Δ
Left and right boundary conditions known
Internal conditions known, but labels $M_i$ unknown

Grey: non linear analytical solution of the Hamilton Jacobi equation. Can be computed explicitly for piecewise affine functions, and semi-explicitly for general nonlinear functions

\[
\begin{align*}
(i) & \quad \inf_{t \in \mathbb{R}_+} \left( g_\gamma(t, \chi) - f_\beta(t, \chi) \right) \geq \Delta \\
(ii) & \quad \Delta \geq \sup_{t \in \mathbb{R}_+} \left( -g_\beta(t, \xi) + f_\gamma(t, \xi) \right) \\
(iii) & \quad \inf_{t \in [t_{\text{min}}, t_{\text{max}}]} \left( g_\gamma(t, \bar{x}_i(t)) \right) \geq M_i \quad \forall i \in I \\
(iv) & \quad M_i \geq \sup_{t \in \mathbb{R}_+} \left( f_\gamma(t, \xi) - g_\beta(t, \xi) \right) \quad \forall i \in I \\
(v) & \quad \inf_{t \in [t_{\text{min}}, t_{\text{max}}]} \left( g_\beta(t, \bar{x}_i(t)) \right) \geq \Delta + M_i \quad \forall i \in I \\
(vi) & \quad M_j - M_i \geq \sup_{t \in [t_{\text{min}}, t_{\text{max}}]} \left( -g_\beta(t, \bar{x}_i(t)) \right) \quad \forall i \in I, \forall j \in I \setminus \{i\}
\end{align*}
\]

Outline

1. Traffic information systems
   1. Existing dedicated traffic monitoring infrastructure
   2. Web 2.0 on wheels
2. Mobile Millennium
   1. System
   2. Privacy aware sampling
3. Inverse modeling and data assimilation
   1. A short introduction to traffic modeling
   2. The Moskowitz Hamilton-Jacobi equation
   3. Internal boundary conditions using the inf-morphism property
   4. Data assimilation in a privacy aware environment
5. Mobile Century (February 8th, 2008)
4. The launch of Mobile Millennium
   1. The Bay Area
   2. New York
Prototype experiment: *Mobile Century*

**Experimental proof of concept: the Mobile Century field test**
- February 8th 2008
- I80, Union City, CA
- Field test, 100 cars
- 165 Berkeley students drivers
- 10 hours deployment,
- About 10 miles
- 2% - 5% penetration rate

**Mobile Century validation video data collection**

**Video data:**
- Vehicles counts
- Travel time validation
A glimpse of Mobile Century (February 8th, 2008)
Mobile Century experimental data

Data assimilation using linear programming

Example: Computation of the upper and lower bounds on the total number of vehicles at initial time:

\[
\begin{align*}
\text{Minimize (respectively Maximize):} \quad & \Delta \\
\text{Subject to:} \\
\inf_{t \in \mathbb{R}_+} (g_x(t,x) - f_x(t,x)) & \geq \Delta \\
\Delta & \geq \sup_{t \in \mathbb{R}_+} (-g_x(t,x) + f_x(t,x)) \\
\inf_{t \in [t_{\min}, t_{\max}]} (g_x(t, \mathcal{F}_i(t))) & \geq \bar{M}_i \quad \forall i \in I \\
\bar{M}_i & \geq \sup_{t \in \mathbb{R}_+} (f_x(t,x) - g_{\mu_i}(t,x)) \quad \forall i \in I \\
\inf_{t \in [t_{\min}, t_{\max}]} (g_x(t, \mathcal{F}_j(t))) & \geq -\Delta + \bar{M}_i \quad \forall i \in I \\
\bar{M}_j - \Delta & \geq \sup_{t \in \mathbb{R}_+} (f_x(t,x) - g_{\mu_j}(t,x)) \quad \forall i \in I, \forall j \in I \setminus \{i\} \\
\bar{M}_j - \bar{M}_i & \geq \sup_{t \in [t_{\min}, t_{\max}]} (-g_{\mu_j}(t, \mathcal{F}_i(t))) \quad \forall i \in I, \forall j \in I \setminus \{i\}
\end{align*}
\]

[Claudel, Bayen, in prep. 2009, SIAM SICON, data assimilation]
Bounds on travel time (PeMS)

Outflow loop
Inflow loop

Bounds on travel time (PeMS and phones)

Outflow loop
Inflow loop
Revealing the previously unobservable

5 car pile up accident (not Mobile Century vehicles)
- Captured in real time
- Delay broadcast to the system in less than one minute

Validation of the data (video)

Travel time predictions
- Can be done in real time at a 2% penetration rate of traffic
- Proved accurate against data from www.511.org, with higher degree of granularity
Mobile Millennium: a pilot project

Mission statement
The goal of Mobile Millennium is to establish the design of a system that collects data from GPS-enabled mobile phones, fuses it with data from existing sensors and turn it into relevant traffic information.

Mobile Millennium is a field operational test
– Deployment of thousands of cars on a network including arterials
– Participating users agree to share position and speed
– Phones receive live information on map application
– Project duration at least 6 months
– Mobile Millennium is a pilot

Launch
Mobile Millennium was launched on November 10th, at 8:30am from the UC Berkeley campus
Mobile Millennium: a pilot project

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Testing the Mobile Millennium in Manhattan
Testing the *Mobile Millennium* in Manhattan

Inter VTL travel time
Each bar is one record of one anonymous vehicle
Test: October 24th, 2008

Arterial network covered by Mobile Millennium

**Arterial model**
Category 1 to 4 roads running in Mobile Millennium. Fuse VTL data, historical NAVTEQ data), taxi data.