Modal Models in Ptolemy

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Online

Online interactive versions of most of the examples in this talk can be accessed by clicking on the figures in the paper listed here:

http://www.eecs.berkeley.edu/Pubs/TechRpts/2009/EECS-2009-151.html
Influences for This Work

- Statecharts [Harel 87]
- Argos [Maraninchi 91]
- Esterel [Berry & Gonthier 92]
- Abstract state machines [Gurevich 93]
- Hybrid systems [Puri & Varaiya 94, Henzinger 99]
- Timed automata [Alur & Dill 94]
- SyncCharts [Andre 96]
- I/O Automata [Lynch 96]
- *Charts [Girault, Lee, & Lee 99]
- UML State machines

What does this have to do with Equational Models?

Fixed-point semantics for a rich variety of concurrent models of computation:

- Fixed-point semantics: $s = F(s)$
- Dataflow models: $s$ is a tuple of sequences.
- Synchronous/reactive models: $s$ is a tuple of values, and $F = F_i$ varies in each tick $i$.
- Discrete-event models: SR model where each tick occurs at a time $t_i$ in a model of time.
- Continuous-time models: DE models where a solver chooses the values of $t_i$.

[Lee & Zheng, EMSOFT 07]
What does this have to do with Equational Models?

For SR, DE, and CT models, the function $F$ changes over time.

This talk gives a semantics to these changes using the notion of modal models.

A major goal is a clean semantics for hybrid systems.

Motivating Example: Hybrid System

Finite State Machine

Guard: $abv/\text{wheel}>\text{Stickiness}

Set: $\text{Stickiness} = P1$

Guard: $\text{true}$

Output: $P1 = p1$, $P2 = p2$

Guard: $\text{true}$

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Meta Model for FSMs in Ptolemy II

- Initial state indicated in bold
- Guards are expressions that can reference inputs and variables
- Output values can be functions of inputs and variables
- Transition can update variable values ("set" actions)
- Final state terminates execution of the actor

Extended State Machines
Reference and manipulate variables on guards and transitions.

Extended state machines can operate on variables in the model, like "count" in this example.

"Set" actions are distinct from "output" actions. We will see why.
Modal Model Meta Model
Hierarchical Composition

An actor’s behavior may be defined by an arbitrarily deep nesting of FSMs and refinements.

Director determines semantics of the submodel

Ptolemy II Enables Hierarchical Mixtures of MoCs

This model has two simple synchronous/reactive (SR) models as mode refinements and models their timed environment using a discrete-event (DE) director.
AND states

Here, two FSMs are composed under a synchronous/reactive director, resulting in Statecharts-like AND states.

Using a synchronous/reactive (SR) director yields Statechart-like semantics for concurrent state machines.

Operational Semantics: Firing

An actor’s behavior may be defined by an arbitrarily deep nesting of FSMs and refinements.
Operational Semantics: Postfiring

State changes are committed only in postfire, enabling fixed point iteration by using only firing.

Fixed-Point Semantics of SR is Enabled by Fire/Postfire Separation

Result is a constructive semantics.

The example here requires multiple firings of the FSM actors to converge to the least fixed point.
Directors Benefiting from Fire/Postfire Separation (which we call the Actor Abstract Semantics)

- **Synchronous/Reactive (SR)**
  - Execution at each tick is defined by a least fixed point of monotonic functions on a finite lattice, where bottom represents "unknown" (getting a constructive semantics)

- **Discrete Event (DE)**
  - Extends SR by defining a "time between ticks" and providing a mechanism for actors to control this. Time between ticks can be zero ("superdense time").

- **Continuous**
  - Extends DE with a "solver" that chooses time between ticks to accurately estimate ODE solutions, and fires all actors on every tick.

[Lee & Zheng, EMSOFT 07]

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The Modal Model Muddle

It's about time

After trying several variants on the semantics of modal time, we settled on this:

A mode refinement has a *local* notion of time. When the mode refinement is inactive, local time does not advance. Local time has a monotonically increasing gap relative to global time.
Modal Time Example

DiscreteClock generates regularly spaced events that trigger mode transitions.

These transitions are "history" transitions, so mode refinements preserve state while suspended.

Produce regularly spaced events in this mode.

Modal Time Example

Mode transitions triggered at times 0, 2.5, 5, 7.5, etc.

Events with value 1 produced at (local times) 0, 1, 2, 3, etc.

First regular event generated at (global time) 0, then transition is immediately taken. First irregular event generated at (global time) 0, one tick later (in superdense time).

Spontaneous Modal Model

Local time 1 corresponds to global time 3.5 here.
Variant using Preemptive Transition

First regular event is not produced until global time 2.5 (local time 0).

Preemptive transition

Time Delays in Modal Models

Triggers transitions at (global times) 0, 1, 2, 3, …

First output is the second input to the modal model, which goes through the noDelay refinement.

Second output is the first input to the modal model, which goes through the delay refinement, which is inactive from time 0 to 1.
Variants for the Semantics of Modal Time that we Tried or Considered, but that Failed

- Mode refinement executes while “inactive” but inputs are not provided and outputs are not observed.
- Time advances while mode is inactive, and mode refinement is responsible for “catching up.”
- Mode refinement is “notified” when it has requested time increments that are not met because it is inactive.
- When a mode refinement is re-activated, it resumes from its first missed event.

_All of these led to some very strange models…_

_Final solution: Local time does not advance while a mode is inactive. Growing gap between local time and global time._

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Formalization (Detailed in the Paper)

- **Actor:**
  \[ A = (S, s_0, I, O, F, P) \]

- **Output function (“fire”):**
  \[ F : S \times I \times N \rightarrow O \]

- **State-update function (“postfire”):**
  \[ P : S \times I \times N \rightarrow S \]
Semantics – untimed

- Set of traces:
  \[ s_0 \xrightarrow{x_0, y_0} s_1 \xrightarrow{x_1, y_1} s_2 \xrightarrow{x_2, y_2} \ldots \]
  such that for all \( i \), there exists \( j \), s.t.:
  \[
  y_i = F(s_i, x_i, j) \\
  s_{i+1} = P(s_i, x_i, j)
  \]

Semantics – timed

- States include special *timer* variables:
  - Can be suspended (“frozen”, inactive) and resumed (active)
  - Expire when they reach 0
- Set of timed traces:
  \[ s_0 \xrightarrow{x_0, y_0, d_0} s_1 \xrightarrow{x_1, y_1, d_1} s_2 \xrightarrow{x_2, y_2, d_2} \ldots \]
  such that for all \( i \), there exists \( j \), s.t.:
  \[
  y_i = F(s_i, x_i, j) \\
  s_{i+1} = P(s_i - d_i, x_i, j) \\
  d_i \leq \min \{ v \mid v \text{ is the value of an active timer in } s_i \}
  \]
Composition and heterogeneity

- Modular semantics:
  - Given a composite actor A, with sub-actors A1, A2, …
  - the F and P functions for A are defined from the Fi, Pi, functions of sub-actors Ai.

- How F and P are defined depends on the director used in A
  - Directors = composition operators

- Heterogeneity:
  - Different directors implement different composition models

Giving semantics to modal models

Goal:
define $F, P$ functions for the modal model

$F_1, P_1$ functions already defined for this refinement

$F_2, P_2$ functions already defined for the "controller" automaton

$F_3, P_3$ functions already defined for this refinement
Rough description of semantics

- Given current controller state $s_i$:
  - If no outgoing transitions from $s_i$ are enabled:
    - Use $F_i$ and $P_i$ to compute $F$ and $P$
  - If preemptive outgoing transitions from $s_i$ are enabled:
    - Use the actions of these transitions to compute $F$ and $P$
  - If only non-preemptive outgoing transitions from $s_i$ are enabled:
    - First fire refinement, then transition, i.e.:
      - $F$ is the composition of $F_i$ and the output action of a transition
      - $P$ is the composition of $P_i$ and the state update action of a transition
- Timers of refinements suspended and resumed when exiting/entering states
- Details in the paper

More Variants of Modal Models Supported in Ptolemy II

- Transition may be a reset transition
  - Destination refinement is initialized
- Multiple states can share a refinement
  - Facilitates sharing internal actor state across modes
- A state may have multiple refinements
  - Executed in sequence (providing imperative semantics)
Still More Variants

- Transition may have a refinement
  - Refinement is fired when transition is chosen
  - Postfired when transition is committed
  - Time is that of the environment

And Still More Variants

- Transition may be a “default transition”
  - Taken if no non-default transition is taken
  - Compare with priorities in SyncCharts
- FSMs may be nondeterminate
  - Can mark transitions to permit nondeterminism
Conclusion

Modal models (in Ptolemy II, Statecharts, SyncCharts, Argos, etc.) provide a hierarchical mixture of imperative logic and declarative composition.

Humans are very capable of reasoning both *imperatively* (algorithms, recipes, etc.) and *declaratively* (equations, synchronous composition, etc.). We use these reasoning tools for *different* (complementary) tasks.

Models that support both will eventually replace models that provide only one or the other.

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- **Other:**
  - David Hermann & Zoltan Kemenczy (from RIM): transition refinements
  - Jie Liu: hybrid systems
  - Ye Zhou: modal dataflow models
New Text very relevant to this conference:

Lee & Seshia: 
Introduction to Embedded Systems
- A Cyber-Physical Systems Approach

Electronic edition is available for free here:

http://LeeSeshia.org/

This book has a strong theme of model-based design of embedded systems.

Syntax of AND States
In Statecharts, communication between concurrent state machines is specified by name matching.

Example from Reinhard von Hanxleden, Kiel University

Communication path
Can you tell whether there is feedback?
(This might be called the modal model meta model muddle).
Syntax of AND States
In Ptolemy II, communication between concurrent state machines is explicit.

Now can you tell whether there is feedback?

This is also more modular because names don’t need to match at the source and destination of a connection.

This model turns the pedestrian lights green when the car control lights go red.